Trading QQQ ETF Daily Bars Using the nth Order Fixed Memory Polynomial Velocity Algorithm Walk Forward in-sample 21 Weekdays and out-of-sample 5 Weekdays. 1/04/2021 to 01/03/2025 using The Walk Forward Metric Explorer Working Paper January 2025 Copyright © 2025 Dennis Meyers

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In previous working papers https://meyersanalytics.com/papers we showed how the application of a price curve generated by the **Nth Order Fixed Memory Polynomial Velocity** could be used to develop a strategy to buy and sell futures and stocks intraday. The reason behind this type of strategy was to only trade when the price trend velocity was above a certain threshold. Many times, prices meander around without any notable trend, and this is considered noise. During these times we do not wish to trade because of the cost of whipsaw losses that would occur from this type of price action. When a price trend finally starts, the velocity of that price trend moves above a minimum threshold noise value. Thus, the velocity strategy would only issue a trade when certain velocity thresholds above "noise" levels are crossed.

The velocity strategy that we will use here to trade the Invesco QQQ Trust Series ETF (**QQQ**) is called the nth Order Polynomial Velocity Strategy. The nth Order Adaptive Polynomial Velocity Strategy has several unknown inputs that we must determine before we can use this strategy to trade. These unknown inputs are the polynomial order or degree, the optimum number of lookback prices we need to determine the coefficients of the polynomial and finally the velocity thresholds. Here we will use Walk Forward Optimization and out-of-sample testing to determine the "best" polynomial inputs as well as how these inputs change over time. We will use the nth Order Fixed Memory Polynomial Velocity Strategy to trade the QQQ ETF on a daily basis using daily bar price data from 1/4/2021 to 01/03/2025.

### The n<sup>th</sup> Order Fixed Memory Velocity Strategy Defined

The least squares forecast n<sup>th</sup> order fixed memory polynomial velocity is constructed by solving for the coefficients  $\beta_0$ ,  $\beta_1$ ,  $\beta_2$ ,  $\beta_3$  ...  $\beta_n$  for the discrete orthogonal Legendre polynomials each day using the last **N bars** of closing prices and the equation for  $\beta_j$  shown in the **"Math"** appendix at the end of this working paper. Then n<sup>th</sup> Order Fixed Memory Polynomial **Velocity(T+1)** is constructed from the equation shown in the "Math" appendix.

Due to polynomial mathematics, the Velocity of the 2<sup>nd</sup>, 3<sup>rd</sup> and 4<sup>th</sup> order degree polynomial curve changes faster than its corresponding first order degree polynomial velocity. Whether higher order polynomial velocities are an advantage or not, will be determined by the computer when we use a walk forward optimization technique described below.

At each bar we calculate the nth order degree (1<sup>st</sup> through 3<sup>rd</sup>) fixed memory polynomial velocity from the formulas in the "Math" appendix. As will be shown below, walk forward optimization will determine the **degree** for the nth order polynomial velocity, the number of lookback prices, **N**, needed to compute the polynomial coefficients and the threshold amounts **vup** and **vdn**. When the nth order degree velocity is greater than the threshold amount **vup** we will go long. When the velocity is less than the threshold amount **-vdn** we will go short.

### Buy Rule:

IF Velocity is greater or equal than the amount *vup and Velocity* [1] is less than vup than buy at the market.

### Sell Rule:

IF Velocity is less than or equal than the amount -vdn and Velocity [1] is greater than -vdn than sell at the market.

Where Velocity [1] is the velocity of the previous bar.

### Friday Close Exit Rule:

Close the position on Friday close (No trades will be carried out over the weekend).

### Testing The Polynomial Velocity Strategy Using Walk Forward Optimization

There will be four strategy parameters to determine:

- 1. **degree**, degree=1 for straight line velocity, degree=2 for 2<sup>nd</sup> order velocity, etc.
- 2. **N**, is the number of lookback bars of prices to calculate the **velocity**.
- 3. *vup*, the threshold amount that velocity must be greater than to issue a buy signal.
- 4. *vdn*, the threshold amount that velocity must be less than to issue a sell signal.

As mentioned, to test this Strategy we will use daily bar prices of the Invesco QQQ Trust Series ETF traded on the NYSE, known by the symbol QQQ for the 205 weeks from January 4, 2021, to January 3, 2025. However, The Walk Forward Input Explorer will only analyses data from 1/4/2021 to 7/5/2024. 7/12/24 to 01/05/25 will be withheld to see how the filter discovered in 1/4/21 to 7/5/24 applied to the period of 7/12/2024-1/3/2025 did. This is approximately 6 months of trading days, from 7/12/24 to 01/03/25. Why did we do this? In the WFME output there is a parameter named BE, Break even(BE) in oos periods. Assuming the average and standard deviation are from a normal distribution, this is the number of oos periods you would have to trade to have a 98% probability that your oos equity is above zero. Since most of the BEs in the WFME are less than 50, we wanted to see what % of returns with a BE< 50 would be positive.

We will test the FixmVn strategy with the above QQQ ETF daily bars on a *walk forward basis*, where the in-sample (*IS*) will be 21 trading weekdays, and the out-of-sample (*OOS*) will be the next 5 weekdays following as will be described below.

### What Is Walk Forward Optimization with In-Sample Section and Out-Of-Sample Sections?

Whenever we do a TradeStation or Multicharts (TS/MC) optimization on many different strategy inputs, TS/MC generates a report of performance metrics (total net profits, number of losing trades, etc.) vs these different strategy inputs. If the report is sorted on say the total net profits(*tnp*) performance metric column, then the highest *tnp* would correspond to a certain set of inputs. This is called an *in-sample (IS) section*. If we choose a set of strategy inputs from this report based upon some performance metric, we have no idea whether these strategy inputs will produce the same results on future price data or data they have not been tested on. Price data that is not in the in-sample section is defined as *out-of-sample data*. Since the performance metrics generated in the insample section are mostly due to "curve fitting" or "data mining" it is important to see how the strategy inputs chosen from the in-sample section perform on out-of-sample data.

What do we mean by *"curve fitting" or data mining?* As a simple example, suppose you were taking the subway to work. In the subway car you are in, suppose you counted the number of blond women in that car and suppose the percent of blond women vs all other women hair colors was 80%. Being that you cannot observe what is in the other subway cars, you would assume that all the other subway cars and perhaps all women had the same percentage of blond hair. This observation was due to chance. That is an example of curve fitting. The same goes for combinatorial searches. You are observing results from a finite sample of data without knowing the data outside the sample you examined.

Walk forward analysis attempts to minimize the curve fitting of price noise by using the law of averages from the Central Limit Theorem on the out-of-sample performance. In walk forward analysis the data is broken up into many in-sample and out-of-sample sections. Usually for any strategy, one has some performance metric selection procedure, which we will call a *filter*, used to select the strategy input parameters from the optimization run. For instance, a *filter* example might be all cases that have a profit factor (PF) greater than 1 and less than 3. For the number of cases left, we might select the cases that had the best percent profit. This procedure would leave you with one case in the in-sample section output and its associated strategy input parameters. Now suppose we ran our optimization on each of our many in-sample sections and applied our *filter* to each in-sample section output. We would then use the strategy input parameters found by the *filter* in each in-sample section on the out-of-

sample section immediately following that in-sample section. The input parameters found in each in-sample section and applied to each out-of-sample section would produce independent net profits or losses for each of the out-of-sample sections. Using this method, we now have "x" number of independent out-of-sample section profit and losses from our *filter*. If we take the average of these out-of-sample section net profits and losses, then we will have an estimate of how our strategy will perform on average. Due to the Central Limit Theorem, as the sample size increases, the spurious noise results in the out-of-sample section performance tend to average out to zero *in the limit*, leaving us with what to expect from our strategy and filter. *Mathematical note: This assumption assumes that the out-of-sample returns are from probability distributions that have a finite variance*.

Why use the walk forward technique? Why not just perform an optimization on the whole price series and choose the input parameters that give the best total net profits or profit factor or some other performance metric? Surely the price noise cancels itself out with such a large number of in-sample prices and trades. Unfortunately, nothing could be farther from the truth! Optimization is a misnomer and should really be called combinatorial search. As stated above, whenever we run a combinatorial search over many different combinations of input parameters on noisy data on a fixed number of prices, no matter how many, the best performance parameters found are guaranteed to be due to "curve fitting" the noise and signal. The price series that we trade consists of random spurious price movements, which we call noise, and repeatable price patterns (if they exist). When we run, for example, 5000 different inputs parameter combinations, the best performance parameters will be from those strategy input variables that are able to produce profits from the price pattern **and** the random spurious movements While the price patterns will repeat, the same spurious price movements will not. If the spurious price movements that were captured by a certain set of input parameters were a large part of the total net profits, as they are in real price series, then choosing these input parameters will produce losses when traded on future data. These losses occur because the spurious price movements will not be repeated in the same way. This is why strategy optimization or combinatorial searches, also called back testing, with no out-of-sample testing cause loses when traded in real time from something that looked great in the in-sample section.

To gain confidence that our input parameter selection method or filter, using the optimization output of the insample data, will produce profits, we must test the input parameters we found in the in-sample section on out-ofsample data. In addition, we must perform the in-sample/out-of-sample analysis many times. Why not just do the out-of-sample analysis once or just 10 times? Well just as in Poker or any card game, where there is considerable variation in luck from hand to hand, walk forward out-of-sample analysis give considerable variation in out-ofsample profit "luck". That is, by pure chance we may have chosen some input parameter set that did well in the in-sample section data **and** the out-of-sample section data. To minimize this type of "luck", statistically, we must repeat the walk forward out-of-sample (**OOS**) analysis over many (>50) in-sample/out-of-sample return and a standard deviation of out-of-sample returns which allows us to statistically estimate the expected equity and its range for N out-of-sample periods in the future.

### Finding The FixmVn Strategy Parameters Using Walk Forward Optimization

There are four strategy parameters to find, pw, N, vup, vdn.

For the test data we will run the TS or MC optimization engine on **QQQ** daily price bars from 01/04/2021 to 01/03/25 with the following optimization ranges for the FixmVn strategy inputs. This will create **210**, **21 weekday in-sample periods**, **each followed by a 5 weekday out-of-sample period** (See Figure 1 for the in-sample/out-of-sample periods). The days are weekdays only. Weekdays when the OOS falls on an exchange holiday or partial days are eliminated. Holidays that fall on a weekday create a 20-day *IS*. All other *IS* periods consist of 21 trading weekdays.

The optimization ranges are:

- 1. pw=degree from 1 to 3
- 2. N from 5 to 20 in steps of 1.
- 3. vup from 0.25 to 3.5 steps of 0.25
- 4. vdn from 0.25 to 3.5 in steps of 0.25

### 5. Mult = 1.73, iNorm=1 (See Appendix 3, the Normalization Multiplier)

The above pw, n, vup, vdn will produce 9408 different input combinations or cases of the strategy input parameters for each of the 205 in-sample/out-of-sample files for the 4 years of daily bar QQQ data. *Notice* I've included the 2022 period where QQQ had a 20% decline and the 2023, 2024 periods where QQQ had 20% rally's,

### Finding the Best in-sample Metric Filter.

The question we are attempting to answer statistically is which performance metric or combination of performance metrics (which we will call a *filter*) applied to the in-sample section will produce in-sample strategy inputs that produce statistically valid average profits in the out-of-sample section. In other words, we wish to find a performance metric *filter* that we can apply to the in-sample section that can give us strategy inputs that will produce, on average, good trading results in the future.

When TradeStation(TS) or MultiCharts(MC) does an optimization over many combinations of inputs, it creates an output page that has as its rows each strategy input combination and as it's columns various trading performance measures such as Profit Factor, Total Net Profits, etc. An example of a simple filter would be to choose the strategy input optimization row in the in-sample section that had the highest Net Profit or perhaps a row that had the best Profit Factor with their associated strategy inputs. Unfortunately, it was found that this type of simple metric performance filter very rarely produces good out-of-sample results. More complicated metric filters can produce good out-of-sample results minimizing spurious price movement biases in the in-sample selection of strategy inputs.

The combination metric filters are found by a program called WFME64v8x. Details of this program can be found at <u>https://meyersanalytics.com/wfme.html</u>.

All PWFO file metrics used by the WFME64v8x are described at <u>https://meyersanalytics.com/Walk-Forward-Optimization.html</u>.

We will use the WFME program to find one in-sample combination-metric filter applied to each in-sample section which gives a set of strategy inputs which are then applied to each following out-of-sample sections. This will consist of 179 in-sample and out-of-sample sections From 1/04/2021 to 7/5/2024. **Note**: the in-sample sections from 7/12/24-01/03/25 were withheld from the WFME search for the best in-sample metric filter to test if that filter works on data not seen.

Here is a metric combination *filter* found by the WFME64 v8x program that was used in this paper. High profit factors (**pf**) in the in-sample section usually mean poor performance in the out-of-sample-section. This is a kind of reversion to the mean. So, in the in-sample(*IS*) section we eliminate all strategy input rows that have a **pf>5**.. Using the **pf** elimination screen, as described, there can still be 100's of rows left in the in-sample section. The PWFO generates the performance metric named **m(p-rd)**. This metric, **m(p-rd) = Median of All Trades(Final Trade Profit Minus Maximum rundown of Trade).** This statistic measures the difference between the final profit of each trade and the maximum trade loss (rundown) of the trade. The farther the final trade profit is from the maximum trade drawdown, the better the performance of the input variable. Thus, we would want the median to be as large as possible. We use the median for this statistic, because we do not want the statistic distorted by a few outlier trades

.Let us choose the 20 rows in the in-sample section that contain the **maximum m(p-rd)** values from the rows that are left from the **pf** screen. In other words, we sort **m(p-rd)** from high to low, eliminate the rows that have **pf>5** and then choose the largest **top** 20 rows of whatever is left. This filter will now leave 20 cases or rows in the in-sample section that satisfy the above filter conditions. We call this filter **t20m(p-rd)** |**p≤5** where **t20m(p-rd)** means the top or maximum **20 m(p-rd)** rows left *after* the **pf>5** in-sample row elimination. Suppose for this filter, within the 20 in-sample rows that are left, we want the row that has the largest value of the metric called **tWb|tLb**. **tWb|tLb** is the ratio of Total Winning Bars to Total Losing Bars.

We abbreviate this final filter as **t20m(p-rd)** |**p**≤**5-tWb**|**tLb**. For each in-sample section this filter leaves only one row in the in-sample section with its associated strategy inputs and following out-of-sample net profit in the outof-sample section using the strategy inputs found in the in-sample section. This **t20m(p-rd)** |**p**≤**5-tWb**|**tLb** filter is then applied to each of the 179 in-sample sections which give 179 sets of strategy inputs that are used to produce the corresponding 179 out-of-sample performance results. The average out-of-sample performance is calculated from these 179 out-of-sample performance results. In addition, many other important out-of-sample performance statistics for this filter are calculated and summarized.

**Figure 2** and below shows such a computer run along with a small sample of other WFME filter combinations that are constructed in a similar manner. **Row 3** of the sample output in **Figure 2** and below shows the results of the filter discussed above.

From this run, we chose the filter on row 3 of the Figure below. That is, t20m(p-rd) |p≤5-tWb|tLb.

	A	В	С	D	E	F	G	Н	1	J	K	L	M	N	0	P	Q	R	S	Т	U	V	W	Х	Y	Z /
1	QQQdFixmVoxdd21x5td	s02/05/21 e	07/05/24	#179 A	\nyTnp	#26					ISnt2			a0.4	s35.1						c=\$4					
2	Filter-Metric	toGP to	DNP	aoGP a	oTr	ao#T	#	std	skew	kur	t	oW ol	%Wtr	%Р	LLtr	LLp	eqDD	wpr	lpr v	/20	acc	KTau	eqR2	Blw	BE	tkr   bl
3	t20m(p-rd) pf<5 -tWb tLb	30065	29489	242	208.8	1.2	124	668	0.049	2.63	4.04	1.73	58	60	-1749	-1287	-3395	6	6	72	0.36	91	96	14	46	5486
4	t20m(p-rd) pf<5 lr<3-tWb tLb	30065	29489	242	208.8	1.2	124	668	0.049	2.63	4.04	1.73	58	60	-1749	-1287	-3395	6	6	72	0.36	91	96	14	46	5486
5	t20m(p-rd) pf<5 lr<5-tWb tLb	30065	29489	242	208.8	1.2	124	668	0.049	2.63	4.04	1.73	58	60	-1749	-1287	-3395	6	6	72	0.36	91	96	14	46	5486
6	t20m(p-rd) pf<5 lr<5-mLb	30928	30396	267	232.5	1.1	116	673	-0.152	3.15	4.26	1.58	62	66	-1749	-1723	-2426	8	5	44	-0.07	93	96	18	41	5112
7	t20m(p-rd) pf<5 lr<3-mLb	30928	30396	267	232.5	1.1	116	673	-0.152	3.15	4.26	1.58	62	66	-1749	-1723	-2426	8	5	44	-0.07	93	96	18	41	5112
8	t20m(p-rd) pf<5 -mLb	30928	30396	267	232.5	1.1	116	673	-0.152	3.15	4.26	1.58	62	66	-1749	-1723	-2426	8	5	44	-0.07	93	96	18	41	5112
9	t20m(p-rd) pf<4 lr<3-tWb tLb	28090	27502	223	191.1	1.2	126	693	0.043	2.75	3.61	1.61	57	59	-1749	-1568	-2923	5	5	78	0.92	90	96	12	58	4524
10	t20m(p-rd) pf<4 lr<5-tWb tLb	28090	27502	223	191.1	1.2	126	693	0.043	2.75	3.61	1.61	57	59	-1749	-1568	-2923	5	5	78	0.92	90	96	12	58	4524
11	t20m(p-rd) pf<4 -tWb tLb	28090	27502	223	191.1	1.2	126	693	0.043	2.75	3.61	1.61	57	59	-1749	-1568	-2923	5	5	78	0.92	90	96	12	58	4524
12	t20m(p-rd) pf<5 -tnp	27253	26669	215	186.7	1.1	127	707	-0.323	3.6	3.42	1.5	58	61	-1749	-2259	-2704	8	5	38	0.23	92	97	12	64	3959
13	t20m(p-rd) pf<5 lr<5-tnp	27253	26669	215	186.7	1.1	127	707	-0.323	3.6	3.42	1.5	58	61	-1749	-2259	-2704	8	5	38	0.23	92	97	12	64	3959
14	t20m(p-rd) pf<5 lr<3-tnp	27253	26669	215	186.7	1.1	127	707	-0.323	3.6	3.42	1.5	58	61	-1749	-2259	-2704	8	5	38	0.23	92	97	12	64	3959
15	t20m(p-rd) pf<5 -tLb	29437	28933	254	233.6	1.1	116	678	-0.068	2.85	4.03	1.51	62	63	-1749	-1497	-3191	8	6	100	0.49	89	96	20	46	3722
16	t20m(p-rd) pf<5 lr<5-tLb	29437	28933	254	233.6	1.1	116	678	-0.068	2.85	4.03	1.51	62	63	-1749	-1497	-3191	8	6	100	0.49	89	96	20	46	3722
17	t20m(p-rd) pf<5 lr<3-tLb	29437	28933	254	233.6	1.1	116	678	-0.068	2.85	4.03	1.51	62	63	-1749	-1497	-3191	8	6	100	0.49	89	96	20	46	3722
18	t20m(p-rd) pf<5 lr<5-tWb	28235	27639	222	189.5	1.2	127	708	-0.445	3.65	3.54	1.4	60	64	-1749	-2259	-3395	11	6	15	0.33	90	96	14	60	3643
19	t20m(p-rd) pf<5 lr<3-tWb	28235	27639	222	189.5	1.2	127	708	-0.445	3.65	3.54	1.4	60	64	-1749	-2259	-3395	11	6	15	0.33	90	96	14	60	3643
20	t20m(p-rd) pf<5 -tWb	28235	27639	222	189.5	1.2	127	708	-0.445	3.65	3.54	1.4	60	64	-1749	-2259	-3395	11	6	15	0.33	90	96	14	60	3643
	A	B C	D	E	F	G	H	1	1																	
1	QQQdFixmVoxdd21x5td	s07/12/24	e01/03/2	25 #26			t205	1	57660																	
2	Filter-Metric	toGPx	toNPx	aoTR	tx aoN	Fx #x	tOn	pNe I	Prob	_										_						
3	t20m(p-rd) pf<5 -tWb tLb	5271	5199	9 293	3 1.	1 16	34	688 (	5.89E-12	2 Т	'his i	is the	2 <sup>na</sup>	sec	tion	fror	n 7/:	12/3	202	24 t	0					
4	t20m(p-rd) pf<5 lr<3-tWb tLb	5271	5199	9 293	3 1.	1 16	34	688 (	5.89E-12	20	1/0	3/202	25 w	hic	h wa	as no	ot ind	lud	ed	in	the '	Wal	k			
5	t20m(p-rd) pf<5 lr<5-tWb tLb	5271	5199	9 293	3 1.	1 16	34	688 (	5.89E-12	2 5	-, -	ard N	Actr	ic E	valo	ror		Thi	ic	ho						
6	t20m(p-rd) pf<5 lr<5-mLb	4996	4936	5 333	3 1.	2 13	35	332 4	4.71E-14	1 7	UI W	aru	vieu		xpio	irer	un.	11113	5 15	110	wu	e				
7	t20m(p-rd) pf<5 lr<3-mLb	4996	4936	5 333	3 1.	2 13	35	332 4	4.71E-14	l fi	ilter	, foui	nd b	y tł	ne W	FME	on	the	1/	4/2	021	-				
8	t20m(p-rd) pf<5 -mLb	4996	4936	5 333	3 1.	2 13	35	332 4	4.71E-14	<sup>1</sup> 0	7/5	/24 d	ata.	pe	rfori	med	in th	ne n	ext	t 26	i we	eks.				
9	t20m(p-rd) pf<4 lr<3-tWb tLb	3316	3248	8 195	i 1.	1 15	30	750 2	2.76E-10	) -	.,-,	,	,	р -												
10	t20m(p-rd) pf<4 lr<5-tWb tLb	3316	3248	8 195	i 1.	1 15	30	750 2	2.76E-10	)																
11	t20m(p-rd) pf<4 -tWb tLb	3316	3248	8 195	i 1.	1 15	30	750 2	2.76E-10	)																
12	t20m(p-rd) pf<5 -tnp	4073	4009	9 255	5 1.	1 15	30	678 1	1.20E-09	)																
13	t20m(p-rd) pf<5 lr<5-tnp	4073	4009	9 255	i 1.	1 15	30	678 1	1.20E-09	)																
14	t20m(p-rd) pf<5 lr<3-tnp	4073	4009	9 255	<b>1</b> .	1 15	30	678 1	1.20E-09	)																
15	t20m(p-rd) pf<5 -tLb	4397	4349	9 366	<b>i</b> 1.	1 11	33	282 (	5.67E-1	3																
16	t20m(p-rd) pf<5 lr<5-tLb	4397	4349	9 366	<b>i</b> 1.	1 11	33	282 (	5.67E-1	3																
17	t20m(p-rd) pf<5 lr<3-tLb	4397	4349	9 366	<b>i</b> 1.	1 11	33	282 (	5.67E-1	3																
18	t20m(p-rd) pf<5 lr<5-tWb	5837	5761	1 307	1.	1 17	33	400 3	3.10E-10	)																
19	t20m(p-rd) pf<5 lr<3-tWb	5837	5761	1 307	1.	1 17	33	400 3	3.10E-10	)																
20	t20m(n_rd)(nt<51_tW/h	5837	5761	1 307	1 1	1 17	- 33	400 3	4 10F-10	1																

## **Bootstrap Probability of Filter Results.**

Using modern "Bootstrap" techniques, we can calculate the probability of obtaining our filter's total out-of-sample *net* profits by chance. Here is how the bootstrap technique is applied. Suppose as an example, we have 500 files of in-sample/out-of-sample data. A mirror random filter is created. Instead of picking an out-of-sample net profit (OSNP) from a filter row as before, the mirror filter picks a *random* row's OSNP in each of the 500 files. We repeat this random picking in each of the 500 files 5000 times. Each of the 5000 mirror filters will choose a random row's OSNP of their own in each of the 500 files. At the end, each of the 5000 mirror filters will have 500 *random* OSNP's

picked from the rows of the 500 files. The sum of the 500 random OSNP picks for each mirror filter will generate a random total out-of-sample net profit (toNP) or final random equity. The average and standard deviation of the 5000-mirror filter's different random toNPs will allow us to calculate the chance probability of our above chosen filter's toNP. Thus, given the mirror filter's bootstrap random toNP average and standard deviation, we can calculate the probability of obtaining our chosen filter's toNP by pure chance alone. Figure 3 lists the 5000-mirror filter's bootstrap average for our 179 out-of-sample files of **\$0.4** with a bootstrap standard deviation of **\$35.1**. (Side Note. The average is the average per out-of-sample period. So, the average for the random selection would be the **random-toNP/179** and the average for the filter would be the **filter toNP/#** of OOS periods traded or **29489/124=237.8**. The probability of obtaining our filters average daily net profit **of 237.8** is **6.89x10**<sup>-12</sup> which is **6.76** standard deviations from the bootstrap average. For our filter, in row 3 and above, the expected number of cases that we could obtain by pure chance that would match or exceed **\$237.8** is  $[1-(1-6.89x10^{-12})^{57660}]$  **or approximately 57660\*6.89x10**<sup>-12</sup> = **0.000000397** where **57660** is the total number of different filters we looked at in this run. This number is much less than one, so it is improbable that our result was due to pure chance.

### Results

**Figure 1** presents a graph of the equity curve generated by using the filter on the 179 weeks from 1/4/21 to 7/5/24. Separated by a red line from the data from 26 weeks from 7/12/24 to 01/03/25 that were not included in the WFME filter search. The equity curves are plotted from Equity and Net Equity columns in Table 1. Plotted on the equity curves is the  $2^{nd}$  Order Polynomial curve. The blue line is the equity curve without commissions and the red dots on the blue line are new highs in equity. The brown line is the equity curve with commissions and the green dots are the new highs in net equity. The grey line is the QQQ weekly closing prices superimposed on the Equity Chart.

**Figure 2** presents a plot of the FixmVn Strategy buy/sells and the FixmVn Indicator on the QQQ daily bars from 9/13/24 to 12/06/24. Note since the FixmV inputs change every week depending upon the satisfying the filter, I plotted the weeks where the FixmV inputs changed by very little.

**Table 1** below presents a table of the total 205 in-sample and out-of-sample windows, the **Filter selected** in-sample strategy inputs and the weekly out-of-sample profit/loss results using the filter described above. Plus, the26 weeks from 7/12/24 to 01/03/25 that were not included in the WFME filter that was run from 1/4/21 - 7/5/24.

## **Discussion of Strategy Performance**

In Figure 3, Row 3 of the spreadsheet filter output are some statistics that are of interest for our filter. An interesting statistic is **Blw**. **Blw** is the maximum number of weeks the OSNP equity curve failed to make a new high. **Blw** is **14** weeks for this filter. This means that 14 weeks were the longest time that the equity for this strategy failed to make a new equity high. **%Wtr** is the percentage of all OOS trades that were wins or positive. For this filter, the **%Wtr=58%**. **%P** is the % winning oos weeks, **%P=60%**. The average oos winning trade to the average oos losing trade ratio(**oW|oL**) was **1.73**. **wpr=6** is the maximum number of consecutive winning oos periods(weeks) in a row and **lpr=6** is the maximum number of consecutive losing oos periods(weeks) in a row. The Largest losing trade in the 1/4/21-7/5/24 period was (\$1749) and the largest losing week was (\$1287). The average trade was \$208.8 @ 1.2 average trades on weeks that it traded for the 1/4/21-7/5/24 period and average trade of \$293 @1.1 average trades on weeks that it traded for the 7/12/21-1/3/25 period.

In Figure 1, which presents a graph of the equity curve using the filter on the 179 weeks of out-of-sample data, notice how the equity curve follows the 2<sup>nd</sup> order polynomial trend line with an R<sup>2</sup> of 0.961. The R<sup>2</sup> only dropped to 0.959 for the net equity curve.

Using this filter, the strategy was able to generate \$29489 net equity after commissions of \$0 (many brokers today, don't charge commissions) and roundtrip slippage of \$4 trading 100 QQQ ETF shares for 179 weeks. The filter generated an extra \$5199 net equity between 7/12/24 to 01/3/25, the data that was not included in the WFME

filter run for a total of \$34688 net equity. This period from 1/18/22 to 10/27/23 was a volatile down then up market as can be seen from the QQQ weekly close on the chart. Yet the FixmVn strategy was able to adapt quite well.

In observing Table 1 we can see that this strategy and filter made trades from a low of no trades/week to a high of 4 trades/week with an average of 1.2 trades/week on the weeks it traded.

### **References**

- 1. Efron, B., Tibshirani, R.J., (1993), "An Introduction to the Bootstrap", New York, Chapman & Hall/CRC.
- 2. Morrison, Norman "Introduction to Sequential Smoothing and Prediction", McGraw-Hill Book Company, New York, 1969.

# Figure 1 Graph of FixmVn Strategy Equity Applying the Walk Forward Filter each week on the in-sample section on QQQ daily Bar Prices 1/4/2021 to 7/5/2024 The red vertical line separates the weeks not included in WFME run 7/12/24-1/3/25

Note: The blue line is the equity curve without commissions and the red dots on the blue line are new highs in equity.

The brown line is the equity curve with commissions and the green dots are the new highs in net equity. The grey line is the QQQ Weekly Closing prices superimposed on the Equity Chart.



Figure 2 Walk Forward Out-Of-Sample Performance Summary for Fixed Memory Polynomial Velocity Strategy QQQ daily bars chart from 9/13/24 to 12/06/24. Note since the FixmV inputs change every week depending upon the filter, I plotted the weeks where the FixmV inputs changed by very little.



# Figure 3 Partial output of the Walk Forward Strategy Inputs with Metric Filters (WFME) QQQ ETF daily bars Using the FixmVn Strategy

	А	В	С	D	E	F	G	Н	1	J	K	L	М	Ν	0	Р	Q	R	S	т	U	V	W	х	Y	ZA
1	QQQdFixmVoxdd21x5td	s02/05/21	e07/05/24	#179	AnyTnp	#26					ISnt2		i	a0.4	s35.1						c=\$4					
2	Filter-Metric	toGP	toNP	aoGP	aoTr	ao#T	#	std	skew	kur	t	oW ol	%Wtr	%P	LLtr	LLp	eqDD	wpr	lpr v	v20	acc	KTau	eqR2	Blw	BE	tkr   bl
3	t20m(p-rd) pf<5 -tWb tLb	30065	29489	242	208.8	1.2	124	668	0.049	2.63	4.04	1.73	58	60	-1749	-1287	-3395	6	6	72	0.36	91	96	14	46	5486
4	t20m(p-rd) pf<5 lr<3-tWb tLb	30065	29489	242	208.8	1.2	124	668	0.049	2.63	4.04	1.73	58	60	-1749	-1287	-3395	6	6	72	0.36	91	96	14	46	5486
5	t20m(p-rd) pf<5 lr<5-tWb tLb	30065	29489	242	208.8	1.2	124	668	0.049	2.63	4.04	1.73	58	60	-1749	-1287	-3395	6	6	72	0.36	91	96	14	46	5486
6	t20m(p-rd) pf<5 lr<5-mLb	30928	30396	267	232.5	1.1	116	673	-0.152	3.15	4.26	1.58	62	66	-1749	-1723	-2426	8	5	44	-0.07	93	96	18	41	5112
7	t20m(p-rd) pf<5 lr<3-mLb	30928	30396	267	232.5	1.1	116	673	-0.152	3.15	4.26	1.58	62	66	-1749	-1723	-2426	8	5	44	-0.07	93	96	18	41	5112
8	t20m(p-rd) pf<5 -mLb	30928	30396	267	232.5	1.1	116	673	-0.152	3.15	4.26	1.58	62	66	-1749	-1723	-2426	8	5	44	-0.07	93	96	18	41	5112
9	t20m(p-rd) pf<4 lr<3-tWb tLb	28090	27502	223	191.1	1.2	126	693	0.043	2.75	3.61	1.61	57	59	-1749	-1568	-2923	5	5	78	0.92	90	96	12	58	4524
10	t20m(p-rd) pf<4 lr<5-tWb tLb	28090	27502	223	191.1	1.2	126	693	0.043	2.75	3.61	1.61	57	59	-1749	-1568	-2923	5	5	78	0.92	90	96	12	58	4524
11	t20m(p-rd) pf<4 -tWb tLb	28090	27502	223	191.1	1.2	126	693	0.043	2.75	3.61	1.61	57	59	-1749	-1568	-2923	5	5	78	0.92	90	96	12	58	4524
12	t20m(p-rd) pf<5 -tnp	27253	26669	215	186.7	1.1	127	707	-0.323	3.6	3.42	1.5	58	61	-1749	-2259	-2704	8	5	38	0.23	92	97	12	64	3959
13	t20m(p-rd) pf<5 lr<5-tnp	27253	26669	215	186.7	1.1	127	707	-0.323	3.6	3.42	1.5	58	61	-1749	-2259	-2704	8	5	38	0.23	92	97	12	64	3959
14	t20m(p-rd) pf<5 lr<3-tnp	27253	26669	215	186.7	1.1	127	707	-0.323	3.6	3.42	1.5	58	61	-1749	-2259	-2704	8	5	38	0.23	92	97	12	64	3959
15	t20m(p-rd) pf<5 -tLb	29437	28933	254	233.6	1.1	116	678	-0.068	2.85	4.03	1.51	62	63	-1749	-1497	-3191	8	6	100	0.49	89	96	20	46	3722
16	t20m(p-rd) pf<5 lr<5-tLb	29437	28933	254	233.6	1.1	116	678	-0.068	2.85	4.03	1.51	62	63	-1749	-1497	-3191	8	6	100	0.49	89	96	20	46	3722
17	t20m(p-rd) pf<5 lr<3-tLb	29437	28933	254	233.6	1.1	116	678	-0.068	2.85	4.03	1.51	62	63	-1749	-1497	-3191	8	6	100	0.49	89	96	20	46	3722
18	t20m(p-rd) pf<5 lr<5-tWb	28235	27639	222	189.5	1.2	127	708	-0.445	3.65	3.54	1.4	60	64	-1749	-2259	-3395	11	6	15	0.33	90	96	14	60	3643
19	t20m(p-rd) pf<5 lr<3-tWb	28235	27639	222	189.5	1.2	127	708	-0.445	3.65	3.54	1.4	60	64	-1749	-2259	-3395	11	6	15	0.33	90	96	14	60	3643
20	t20m(p-rd) pf<5 -tWb	28235	27639	222	189.5	1.2	127	708	-0.445	3.65	3.54	1.4	60	64	-1749	-2259	-3395	11	6	15	0.33	90	96	14	60	3643
	A	B C	D	E	E F	G	ł	Н	1																	

1	QQQdFixmVoxdd21x5td	I	s07/12/24	e01/03/25	#26			t205	f57660
2	Filter-Metric	I	toGPx	toNPx	aoTRx	aoNTx	#x	tOnpNe	Prob
3	t20m(p-rd) pf<5 -tWb tLb	I	5271	5199	293	1.1	16	34688	6.89E-12
4	t20m(p-rd) pf<5 lr<3-tWb tLb	I	5271	5199	293	1.1	16	34688	6.89E-12
5	t20m(p-rd) pf<5 lr<5-tWb tLb	I	5271	5199	293	1.1	16	34688	6.89E-12
6	t20m(p-rd) pf<5 lr<5-mLb	I	4996	4936	333	1.2	13	35332	4.71E-14
7	t20m(p-rd) pf<5 lr<3-mLb	I	4996	4936	333	1.2	13	35332	4.71E-14
8	t20m(p-rd) pf<5 -mLb	I	4996	4936	333	1.2	13	35332	4.71E-14
9	t20m(p-rd) pf<4 lr<3-tWb tLb	I	3316	3248	195	1.1	15	30750	2.76E-10
10	t20m(p-rd) pf<4 lr<5-tWb tLb	I	3316	3248	195	1.1	15	30750	2.76E-10
11	t20m(p-rd) pf<4 -tWb tLb	I	3316	3248	195	1.1	15	30750	2.76E-10
12	t20m(p-rd) pf<5 -tnp	I	4073	4009	255	1.1	15	30678	1.20E-09
13	t20m(p-rd) pf<5 lr<5-tnp	I	4073	4009	255	1.1	15	30678	1.20E-09
14	t20m(p-rd) pf<5 lr<3-tnp	I	4073	4009	255	1.1	15	30678	1.20E-09
15	t20m(p-rd) pf<5 -tLb	I	4397	4349	366	1.1	11	33282	6.67E-13
16	t20m(p-rd) pf<5 lr<5-tLb	I	4397	4349	366	1.1	11	33282	6.67E-13
17	t20m(p-rd) pf<5 lr<3-tLb	I	4397	4349	366	1.1	11	33282	6.67E-13
18	t20m(p-rd) pf<5 lr<5-tWb	I	5837	5761	307	1.1	17	33400	3.10E-10
19	t20m(p-rd) pf<5 lr<3-tWb	I	5837	5761	307	1.1	17	33400	3.10E-10
20	t20m(p-rd) pf<5 -tWb	I	5837	5761	307	1.1	17	33400	3.10F-10

## The WFME Filter Output Columns are defined as follows: OOS=out-of-sample

**Row 1** QQQ5Fixmvoxdd21x5tdd is the PWFO output files abbreviation, First OOS Day End Date (2/5/21), Last OOS Day End Date (7/5/24), **Number of weeks** (#179) a=average of bootstrap random picks. **s**= standard deviation of bootstrap random picks. **f**=number of different filters examined. **c**= slippage and round-trip trade cost(c=\$4).

### Row 3 to Last Row Columns: A through Z

Col A: The Strategy Input/Filter Names

Row 3: t20m(p-rd) |p≤5-tWb|tLb: The filter

**Col B:** *toGP* Total out-of-sample(oos) gross profit for these 179 oos periods (for this run period = weeks). **Col C:** *toNP* Total out-of-sample(oos) Net profit (toGP-(# of Trade days)\*cost) for the 179 oos periods.

Col D: *aoGP* Average oos gross profit for the # oos periods

Col E: aoTr Average oos profit per trade

**Col F:** *ao#T* Average number of oos trades per week

**Col G: #** The number of oos periods this filter produced any profit or loss. Note for some oos periods there are no trades.

Col H: std the standard deviation of the # oos period profits and losses

**Col I:** *skew* The Skew statistic of the # oos period profits and losses.

**Col J:** *kur* the kurtosis statistic of the # oos period profits and losses

**Col K:** *t* the student t statistic for the # oos periods. The higher the t statistic the higher the probability that this result was not due to pure chance.

**Col L:** *oW*/*oL* Ratio of Average oos winning trades divided by Average oos losing trades.

**Col M:** *%Wtr* The percentage if oos winning trades.

**Col N:** *%P* percent of all oos periods that were profitable.

**Col O** *LLtr* the largest losing oos trade in all oos periods

Col P: LLp the largest losing oos period

Col Q: eqDD the oos equity drawdown

**Col R:** *wpr* the largest number of winning oos periods (weeks) in a row.

**Col S:** *lpr* the largest number of losing oos periods in a row.

There can be no strategy inputs that satisfy a given filter criteria, and no trades will be made during that period.

**Col T:** *v20 the* straight-line trend of the oos equity curve for the last 20 bars.

**Col U:** *v20 the* straight-line trend of the oos equity curve for the last 20 bars.

**Col V:** *KTau* The Kendall rank coefficient is often used as a test statistic in a statistical hypothesis test to establish whether two variables may be regarded as statistically dependent. This test is non-parametric, as it does not rely on any assumptions on the distributions of X or Y or the distribution of (X,Y)

**Col W:** *eqR2* the correlation coefficient(R^2) of a straight line fit to the equity curve.

**Col X:** *Blw* The maximum number of oos periods the oos equity curve failed to make a new high.

**Col Y: BE** Break even in oos periods. Assuming the average and standard deviation are from a normal distribution, this is the number of oos periods you would have to trade to have a 98% probability that your oos equity is above zero.

**Col Z:** *tkr/bl* =100\*t\*Ktau\*eqR2/(BL\*BE). This is a measure of the best equity curve.

# The Following columns are the results from 7/12/24-1/3/25 that were not included in the filter scan from 1/4/21 to 7/5/24.

**Col AB:** toGPx Total gross profit for the 26 excluded periods (for this run periods = weeks).

**Col AC:** *toNPx* Total Net profit for the 26 excluded periods.

**Col AD:** *aoTrx* Average profit per trade for the 26 excluded periods

**Col AE:** *aoNTx* Average number of trades per week for the 26 excluded periods

**Col AF:** *#x* the number of the 26 excluded periods this strategy/filter traded. Note for some periods there can be no strategy metrics that satisfy the Strategy filter criteria, and no trades will be made during that period.

**Col AG:** *tOnpNet* - toNP+toNPx = Total Net Profits of oos+future periods

Col AH: Prob The probability that the filters OOS toNP from 1/4/21-7/5/24 was due to pure chance

# Table 1 Walk Forward Out-Of-Sample Performance Summary for the QQQ 5-min FixmVn Strategy

QQQ-daily 1/4/2021 - 1/3/2025. Note: The first In-sample section was from 1/1/21-1/29/21 with the first outof-sample section from 02/01/21-02/05/21.

Filter: t20m(p-rd)|pf<5|-twb|tlb; The inputs found by the filter t20m(p-rd)|pf<5|-twb|tlb are used to trade in the following out-of-sample sections.

- **osnp** = Daily out-of-sample gross profit in \$
- **NOnp\$4** = Daily out-of-sample net profit in \$ = **osnp-ont\*4**.

**ont** = The number of trades in the out-of-sample day

**ownp** = winning profits in the out-of-sample day.

**ownt** = number of winning trades in the out-of-sample day

**ollt** = The largest losing trade in the out-of-sample day in \$.

**odd** = The drawdown in the out-of-sample day in \$.

**EQ=Equity** = Running Sum of daily out-of-sample gross profits \$

**NetEq=Net Equity** = running sum of the daily out-of-sample net profits in \$

Note: OSNP rows with 0.0 indicate that no out-of-sample trades were made that week

OOS Dates		pw	N	vup	vdn	osnp	NOnp\$4	ont	ownp	ownt	ollt	odd	EQ	NetEq
02/01/21	02/05/21	2	7	0.5	1.5	154.0	150.0	1	154	1	0	0	154	150
02/08/21	02/12/21	3	9	2.25	0.75	0.0	0.0	0	0	0	0	0	154	150
02/15/21	02/19/21	2	11	1.5	0.5	0.0	0.0	0	0	0	0	0	154	150
02/22/21	02/26/21	3	5	2	1.5	286.0	282.0	1	286	1	0	0	440	432
03/01/21	03/05/21	3	12	1.75	0.75	(188.0)	(192.0)	1	0	0	-188	-188	252	240
03/08/21	03/12/21	2	9	0.5	1.25	(31.0)	(35.0)	1	0	0	-31	-31	221	205
03/15/21	03/19/21	1	6	1.25	0.25	(795.0)	(799.0)	1	0	0	-795	-795	-574	-594
03/22/21	03/26/21	1	6	1.75	0.25	373.0	369.0	1	373	1	0	0	-201	-225
03/29/21	04/02/21	1	9	1.5	0.75	0.0	0.0	0	0	0	0	0	-201	-225
04/05/21	04/09/21	2	6	0.5	3	1254.0	1246.0	2	1254	1	0	0	1053	1021
04/12/21	04/16/21	2	10	0.25	0.75	0.0	0.0	0	0	0	0	0	1053	1021
04/19/21	04/23/21	2	6	0.5	0.75	0.0	0.0	0	0	0	0	0	1053	1021
04/26/21	04/30/21	1	5	0.5	0.5	0.0	0.0	0	0	0	0	0	1053	1021
05/03/21	05/07/21	2	5	0.5	1.75	(17.0)	(21.0)	1	0	0	-17	-17	1036	1000
05/10/21	05/14/21	1	15	1.25	0.25	(669.0)	(673.0)	1	0	0	-669	-669	367	327
05/17/21	05/21/21	1	18	0.75	0.75	(162.0)	(166.0)	1	0	0	-162	-162	205	161
05/24/21	05/28/21	3	17	0.25	1	(14.0)	(18.0)	1	0	0	-14	-14	191	143
05/31/21	06/04/21	2	5	0.75	1.5	0.0	0.0	0	0	0	0	0	191	143
06/07/21	06/11/21	2	5	0.75	1.5	284.0	280.0	1	284	1	0	0	475	423
06/14/21	06/18/21	2	5	0.75	1.5	0.0	0.0	0	0	0	0	0	475	423
06/21/21	06/25/21	1	15	0.5	1.75	531.0	527.0	1	531	1	0	0	1006	950
06/28/21	07/02/21	1	16	0.5	0.25	511.0	507.0	1	511	1	0	0	1517	1457
07/05/21	07/09/21	3	8	0.25	0.75	(144.0)	(148.0)	1	0	0	-144	-144	1373	1309
07/12/21	07/16/21	1	7	0.75	0.75	0.0	0.0	0	0	0	0	0	1373	1309
07/19/21	07/23/21	1	7	0.75	0.75	0.0	0.0	0	0	0	0	0	1373	1309
07/26/21	07/30/21	1	7	0.75	0.75	(365.0)	(369.0)	1	0	0	-365	-365	1008	940
08/02/21	08/06/21	2	7	0.5	2	(32.0)	(36.0)	1	0	0	-32	-32	976	904
08/09/21	08/13/21	1	15	0.25	1.75	(44.0)	(48.0)	1	0	0	-44	-44	932	856
08/16/21	08/20/21	3	9	0.25	2	158.0	154.0	1	158	1	0	0	1090	1010
08/23/21	08/27/21	1	5	0.25	0.75	220.0	216.0	1	220	1	0	0	1310	1226
08/30/21	09/03/21	1	20	0.25	0.75	106.0	102.0	1	106	1	0	0	1416	1328
09/06/21	09/10/21	3	15	0.5	1.75	0.0	0.0	0	0	0	0	0	1416	1328
09/13/21	09/17/21	3	20	0.25	0.5	403.0	399.0	1	403	1	0	0	1819	1727
09/20/21	09/24/21	3	20	1	0.5	(575.0)	(579.0)	1	0	0	-575	-575	1244	1148
09/27/21	10/01/21	2	8	1.75	0.25	99.0	95.0	1	99	1	0	0	1343	1243

10/40/21         10/08/21         3         6         0.5         0.25         671.0         663.0         2         708         1         -37	OOS Dates		pw	Ν	vup	vdn	osnp	NOnp\$4	ont	ownp	ownt	ollt	odd	EQ	NetEq
IDV1121         10/15/21         2         8         0.5         2.5         0.0         0.0         0	10/04/21	10/08/21	3	6	0.5	0.25	671.0	663.0	2	708	1	-37	-37	2014	1906
10/18/21         10/22/21         3         5         0.5         2.5         (184.0)         (18.0)         0         0.0         0         0         0         1380         1718           11/0/211         11/105/21         3         16         0.75         1.5         900.0         1         905         1         0         0         1830         1718           11/0/221         11/12/11         1         16         0.25         1672.0         10         0         0         0         1863         1739           11/12/11         1         16         1.25         0.5         (692.0)         1         0         0         0         0         0         1172         1044           11/16/11         1.1         1.0         0         0         1.55         1.56         11.0         0         0         1.55         1.56         1.66         1.1         0         0         1.55         1.56         1.56         1.56         1.56         1.56         1.56         1.56         1.56         1.56         1.56         1.56         1.56         1.56         1.56         1.56         1.56         1.56         1.56         1.56         1.	10/11/21	10/15/21	2	8	0.5	3.25	0.0	0.0	0	0	0	0	0	2014	1906
ip/25/21         is/25/21         is/25/21         is/2	10/18/21	10/22/21	3	5	0.5	2.5	(184.0)	(188.0)	1	0	0	-184	-184	1830	1718
11/01/21         3         16         0.75         1.5         990.0         91.0         1         995         1         0         0         6         9172         16183         1739           11/15/21         11/12/11         1         16         1.25         1         0.0         0.0         0	10/25/21	10/29/21	3	16	0.75	1.5	0.0	0.0	0	0	0	0	0	1830	1718
$ \begin{array}{c} 11/62/1 \\ 11/15/21 \\ 11/1$	11/01/21	11/05/21	3	16	0.75	1.5	905.0	901.0	1	905	1	0	0	2735	2619
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	11/08/21	11/12/21	2	11	0.5	0.25	(872.0)	(880.0)	2	0	0	-659	-872	1863	1739
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	11/15/21	11/19/21	1	16	1.25	1	0.0	0.0	0	0	0	0	0	1863	1739
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	11/22/21	11/26/21	1	12	0.25	0.5	(691.0)	(695.0)	1	0	0	-691	-691	1172	1044
	11/29/21	12/03/21	1	7	1	1	0.0	0.0	0	0	0	0	0	1172	1044
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	12/06/21	12/10/21	3	8	1	0.5	(294.0)	(302.0)	2	0	0	-155	-294	878	742
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	12/13/21	12/17/21	1	8	1.5	0.5	(158.0)	(162.0)	1	0	0	-158	-158	720	580
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	12/20/21	12/24/21	1	6	1.25	0.25	(1287.0)	(1291.0)	1	0	0	-1287	-1287	-567	-711
01/03/22         01/01/22         0         0         0         0         0         0         0         4404         560           01/01/22         01/14/22         3         13         1.75         1         0.0         0.0         0         0         0         0         4404         560           01/01/22         01/14/22         3         12         1         1         0.0         0.0         0         0         0         4404         560           01/01/22         01/21/22         3         12         1         1         0.0         0.0         0         0         0         1406         1246           01/01/22         02/14/22         13         1.75         1440.0         1136.0         1         144         1         0 <t< td=""><td>12/27/21</td><td>12/31/21</td><td>3</td><td>14</td><td>2</td><td>0.5</td><td>(93.0)</td><td>(101.0)</td><td>2</td><td>0</td><td>0</td><td>-93</td><td>-93</td><td>-660</td><td>-812</td></t<>	12/27/21	12/31/21	3	14	2	0.5	(93.0)	(101.0)	2	0	0	-93	-93	-660	-812
01/10/22         01/14/22         3         13         1.75         1.5         100         0	01/03/22	01/07/22	2	6	2.75	1.25	256.0	252.0	1	256	1	0	0	-404	-560
1         1	01/10/22	01/14/22	2	13	1 75	1	0.0	0.0	0	0	0	0	0	-404	-560
01/01/21         01/01/28/02         0	01/17/22	01/21/22	3	13	1 75	15	1810.0	1806.0	1	1810	1	0	0	1406	1246
$ \begin{array}{c} 2d_12d_12 \\ 0d_13d_122 \\ 0d_12d_122 \\ 0d_12d_122 \\ 0d_12d_122 \\ 0d_12d_122 \\ 0d_1d_122 \\ 0d_1d_1d_122 \\ 0d_1d_122 \\ 0d_1d_122 \\ 0d_1d_122 \\ 0d_1d_122 \\ 0d$	01/24/22	01/22/22	3	12	1.75	1.5	1010.0	1000.0		1010	0	0	0	1/06	1246
02/07/22         03/07/22         03/07/22         03/07/22         03/07/22         03/07/22         03/07/22         03/07/22         03/07/22         03/07/22         03/07/22         03/07/22         03/07/22         03/07/22         03/07/23         03/07/22         03/07/23	01/31/22	01/20/22	1	12	1 25	15	642.0	638.0	1	6/2	1	0	0	20/18	188/
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	02/07/22	02/04/22	2	7	1.25	1.5	600.0	696.0	1	690	1	0	0	2040	2570
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	02/07/22	02/11/22	2	7	2	1 75	11/0 0	1126.0	1	11/0	1	0	0	2730	2370
02/21/22         02/21/22         02/21/22         02/21/22         02/21/22         02/21/22         02/21/22         02/21/22         02/21/22         03/01/22         03/01/22         03/01/22         03/01/22         03/01/22         03/11/22         04/01/22         01/11/23         01/15         02/25         02/25         13/25         13/25         1         0         0         0         0         0         0         0         0         0         0<	02/14/22	02/18/22	1	10	1 75	1.75	1140.0	1130.0	-	1140		0	0	2070	3700
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	02/21/22	02/25/22	2	19	2.75	1.25	0.0	0.0	0	0	0	0	0	3070	3700
05/07/22         05/14/22         05/14/22         05/14/22         05/14/22         05/14/22         05/14/22         05/14/22         03/14/22         03/14/22         03/14/22         03/14/22         03/14/22         03/14/22         03/14/22         03/14/22         03/14/22         03/14/22         03/14/22         03/14/22         03/14/22         03/14/22         03/14/22         03/14/22         03/14/22         03/14/22         03/14/22         04/01/22         3         13         1.5         2         0.0         0.0         0	02/28/22	03/04/22	2	0 0	3.25	1.75	(25.0)	(20.0)	1	0	0	25	25	30/0	3700
03/14/2         03/15/22         3         15         0.7.5         1         (143.0)         2         1312         1         1743         3400         322           03/21/22         03/25/22         3         6         1.2         3.25         0.0         0.0         0	03/07/22	03/11/22	2	0	3.25	1.5	(35.0)	(39.0)	1	1212	0	-35	-35	3843	3007
05/21/22         05/23/22         3         6         1.2         3         1         0         0         1.24         -24         3322         3333           03/28/22         04/01/22         3         1         1.25         2         0.0         0.0         0 <td< td=""><td>03/14/22</td><td>03/18/22</td><td>3</td><td>19</td><td>0.75</td><td>25</td><td>(437.0)</td><td>(445.0)</td><td>2</td><td>1312</td><td>1</td><td>-1749</td><td>-1749</td><td>3406</td><td>3222</td></td<>	03/14/22	03/18/22	3	19	0.75	25	(437.0)	(445.0)	2	1312	1	-1749	-1749	3406	3222
03/28/21         04/04/22         04/04/22         04/04/22         04/04/22         04/04/22         04/04/22         04/04/22         04/04/22         04/04/22         04/04/22         04/04/22         04/04/22         04/04/22         04/04/22         04/04/22         04/04/22         04/04/22         04/04/22         04/04/22         04/11/22         1         1         0         0         44115         3923           04/11/22         04/25/22         04/25/22         04/25/22         04/11/22         1         15         2.5         1.75         122.0         1         0         0         6636           05/02/22         05/11/22         1         1         0.25         1.75         1577.0         1         1         0         0         7097         6889           05/30/22         06/03/22         1         1         3         0.25         1.75         (427.0)         (431.0)         1	03/21/22	03/25/22	3	6	1.25	3.5	(24.0)	(28.0)	1	0	0	-24	-24	3382	3194
04/04/22         04/15/22         3         1.25         2         0.0         0.0         1         133         1         0         0         0         1         133         1         0         0         1         1         0         0         1         1         1         1         1         1         1         1         1         1         0         0         1         1         0         0         1         1         1         0 </td <td>03/28/22</td> <td>04/01/22</td> <td>3</td> <td>6</td> <td>2</td> <td>3.25</td> <td>0.0</td> <td>0.0</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> <td>3382</td> <td>3194</td>	03/28/22	04/01/22	3	6	2	3.25	0.0	0.0	0	0	0	0	0	3382	3194
04/11/22         04/18/22         2         11         0.75         0.25         7/3.0         7/29.0         1         7/3         1         0         0         4115         39/25           04/18/22         04/22/22         2         11         0.75         0.25         1303.0         1255.0         2         1303         1         0         0         6440         6636           05/02/22         05/06/22         3         10         1.5         1.25         0.0         0.0         0         0         0         66840         6636           05/09/22         05/16/22         1         13         0.25         1.75         1577.0         1573.0         1         1577         1         0         0         8674         8462           05/16/22         05/23/22         05/23/22         1         13         0.25         1.75         (427.0)         (431.0)         1         0         0         427         427         8244         8031           05/16/22         06/17/22         3         5         3.5         1.25         (251.0)         (255.0)         1         481         0         0         -251         427         827         <	04/04/22	04/08/22	3	13	1.25	2	0.0	0.0	0	0	0	0	0	3382	3194
04/18/12         04/25/22         1         1         2         5         1.0         0         0         5418         5218           04/25/22         04/25/22         1         15         2.5         0.25         1422.0         1418.0         1         1422         1         0         0         6840         6636           05/02/22         05/03/22         1         13         0.25         1.5         257.0         253.0         1         257         1         0         0         8644         6636           05/03/22         05/23/22         1         13         0.25         1.75         1577.0         1573.0         1         1577         1         0         0         8674         8462           05/30/22         06/03/22         1         13         0.25         1.75         (427.0)         (431.0)         1         0         0         4477         8247         8031           06/06/22         06/01/22         3         5         3.5         1.25         (25.0)         1         481         1         0         0         9164         8936           06/27/22         07/01/22         3         16         2.5	04/11/22	04/15/22	2	11	0.75	0.25	/33.0	/29.0	1	/33	1	0	0	4115	3923
04/25/22       04/29/22       1       15       2.5       0.25       1422.0       1482.0       1       1422       1       0       0       6840       6636         05/09/22       05/06/22       3       10       1.5       1.25       0.0       0.0       0       0       0       0       6840       6636         05/09/22       05/13/22       1       13       0.25       1.5       257.0       23.0       1       1577       1       0       0       8644       8462         05/30/22       05/03/22       1       13       0.25       1.75       (427.0)       (431.0)       1       0       0       427       4247       8031         06/03/22       06/10/22       3       80.75       0.5       481.0       477.0       1       481       1       0       0       8728       8508         06/13/22       06/17/22       3       16       2.5       1       687.0       683.0       1       687       1       0       0       9164       8936         06/20/22       06/21/22       3       16       2.5       1       637.0       70.0       0       0       0	04/18/22	04/22/22	2	11	2.5	0.25	1303.0	1295.0	2	1303	1	0	0	5418	5218
05/02/22         05/06/22         3         10         1.5         1.25         0.0         0.0         0 <td>04/25/22</td> <td>04/29/22</td> <td>1</td> <td>15</td> <td>2.5</td> <td>0.25</td> <td>1422.0</td> <td>1418.0</td> <td>1</td> <td>1422</td> <td>1</td> <td>0</td> <td>0</td> <td>6840</td> <td>6636</td>	04/25/22	04/29/22	1	15	2.5	0.25	1422.0	1418.0	1	1422	1	0	0	6840	6636
05/09/22         05/16/22         05/16/22         05/16/22         05/16/22         1         0         0         7097         68889           05/16/22         05/20/22         1         13         0.25         1.75         1577.0         1573.0         1         1577         1         0         0         8674         8462           05/30/22         05/27/22         1         13         0.25         1.75         (427.0)         (431.0)         1         0         0         -427         -427         8247         8031           06/06/22         06/01/22         3         8         0.75         0.5         481.0         477.0         1         481         1         0         0         8728         8508           06/20/22         06/21/22         3         16         2.55         1         687.0         1         0         0         -318         -318         846         6614           06/27/22         07/21/22         3         16         2.5         1         (318.0)         (322.0)         1         0         0         -318         8346         6614           06/27/22         07/21/22         2         10         1.75 <td>05/02/22</td> <td>05/06/22</td> <td>3</td> <td>10</td> <td>1.5</td> <td>1.25</td> <td>0.0</td> <td>0.0</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> <td>6840</td> <td>6636</td>	05/02/22	05/06/22	3	10	1.5	1.25	0.0	0.0	0	0	0	0	0	6840	6636
05/16/22         05/20/22         1         13         0.25         1.75         1577.0         1573.0         1         1577         1         0         0         8674         88462           05/23/22         05/27/22         1         11         0.25         1.25         0.0         0.0         0         0         0         0         8674         88462           05/30/22         06/03/22         1         13         0.25         1.75         (427.0)         (431.0)         1         0         0         4277         4227         8237           06/06/22         06/13/22         3         5         3.5         1.25         (251.0)         (255.0)         1         481         1         0         0         8728         8508           06/20/22         06/24/22         3         16         2.25         1         687.0         683.0         1         687         1         0         0         9164         8936           06/20/22         07/01/22         3         16         2.25         1         687.0         683.0         1         687         1         0         0         9164         9410           07/15/22	05/09/22	05/13/22	1	6	0.25	1.5	257.0	253.0	1	257	1	0	0	7097	6889
05/23/22         06/03/22         1         11         0.25         1.25         0.0         0 <td>05/16/22</td> <td>05/20/22</td> <td>1</td> <td>13</td> <td>0.25</td> <td>1.75</td> <td>1577.0</td> <td>1573.0</td> <td>1</td> <td>1577</td> <td>1</td> <td>0</td> <td>0</td> <td>8674</td> <td>8462</td>	05/16/22	05/20/22	1	13	0.25	1.75	1577.0	1573.0	1	1577	1	0	0	8674	8462
05/30/22       06/03/22       1       13       0.25       1.75       (427.0)       (431.0)       1       0       0       -427       -427       8247       8031         06/06/22       06/10/22       3       8       0.75       0.5       481.0       477.0       1       481       1       0       0       8728       8508         06/13/22       06/17/22       3       16       2.25       1       687.0       1       0       0       -251       8477       8253         06/27/22       07/01/22       3       16       2.25       1       (318.0)       (322.0)       1       0       0       -318       -318       8846       8614         07/04/22       07/08/22       2       7       1       0.75       800.0       796.0       1       800       1       0       0       9646       9410         07/11/22       07/15/22       2       10       1.75       1.5       0.0       0.0       0       0       0       1227       11983         08/01/22       07/25/22       1       16       0.75       2       883.0       879.0       1       8883       1       0	05/23/22	05/27/22	1	11	0.25	1.25	0.0	0.0	0	0	0	0	0	8674	8462
06/06/22         06/10/22         3         8         0.75         0.5         448.0         477.0         1         481         1         0         0         8728         8508           06/13/22         06/17/22         3         5         3.5         1.25         (251.0)         (255.0)         1         0         0         -251         .251         8477         8253           06/20/22         06/24/22         3         16         2.25         1         687.0         683.0         1         687         1         0         0         9164         8936           06/27/22         07/01/22         3         16         2.55         1         (318.0)         (322.0)         1         0         0         -318         -318         8846         8614           07/04/22         07/01/22         2         0         0.25         1.5         800.0         796.0         1         800         1         0         0         9646         9410           07/11/22         07/21/22         1         20         0.25         1.5         1693.0         1689.0         1         883         1         0         0         12277         11983	05/30/22	06/03/22	1	13	0.25	1.75	(427.0)	(431.0)	1	0	0	-427	-427	8247	8031
06/13/22         06/17/22         3         5         3.5         1.25         (251.0)         (255.0)         1         0         0         -251         -251         8477         8253           06/20/22         06/24/22         3         16         2.25         1         687.0         683.0         1         687         1         0         0         9164         8936           06/27/22         07/01/22         3         16         2.5         1         (318.0)         (322.0)         1         0         0         -318         846         8614           07/04/22         07/05/22         2         10         1.75         1.5         0.0         0.0         0         0         0         9646         9410           07/15/22         07/25/22         1         20         0.25         1.5         1693.0         1689.0         1         1693         1         0         0         12227         11983           08/01/22         08/05/22         1         16         0.75         2         883.0         879.0         1         883         1         0         0         13110         12821           08/08/22         08/12/22<	06/06/22	06/10/22	3	8	0.75	0.5	481.0	477.0	1	481	1	0	0	8728	8508
06/20/22         06/24/22         3         16         2.25         1         687.0         683.0         1         687         1         0         0         9164         8936           06/27/22         07/01/22         3         16         2.5         1         (318.0)         (322.0)         1         0         0         -318         8846         8614           07/04/22         07/08/22         2         7         1         0.75         800.0         760.0         1         800         1         0         0         9646         9410           07/11/22         07/15/22         2         10         1.75         1.5         0.0         0.0         0         0         0         9646         9410           07/18/22         07/25/22         1         20         0.25         1.5         1693.0         1         888         1         0         0         13318         10294           07/25/22         07/29/22         1         16         0.75         2         883.0         884.0         1         883         1         0         0         13318         13066           08/08/22         08/12/22         08/12/22	06/13/22	06/17/22	3	5	3.5	1.25	(251.0)	(255.0)	1	0	0	-251	-251	8477	8253
06/27/22         07/01/22         3         16         2.5         1         (318.0)         (322.0)         1         0         0         -318         -318         8846         8614           07/04/22         07/08/22         2         7         1         0.75         800.0         796.0         1         800         1         0         0         9646         9410           07/11/22         07/15/22         2         10         1.75         1.5         0.0         0.0         0         0         0         0         9646         9410           07/15/22         07/22/22         1         20         0.25         1.5         1693.0         1689.0         1         1693         1         0         0         12227         11983           08/01/22         08/05/22         1         16         0.75         2         883.0         879.0         1         883         1         0         0         13110         12862           08/08/22         08/19/22         3         9         0.25         1.75         (920.0)         (924.0)         1         0         0         13181         13066           08/22/22         08/02/22<	06/20/22	06/24/22	3	16	2.25	1	687.0	683.0	1	687	1	0	0	9164	8936
07/04/22         07/08/22         2         7         1         0.75         800.0         796.0         1         800         1         0         0         9646         9410           07/11/22         07/15/22         2         10         1.75         1.5         0.0         0.0         0         0         0         0         0         9646         9410           07/18/22         07/22/22         1         20         0.25         1.5         1693.0         1888.0         1         0         0         10534         10294           07/25/22         07/29/22         1         16         0.75         2         883.0         879.0         1         883         1         0         0         13110         12862           08/01/22         08/05/22         1         16         0.75         2         208.0         204.0         1         208         1         0         0         13118         13066           08/08/22         08/02/22         08/12/22         3         9         0.25         1.5         00.0         0         0         0         1318         13066           08/22/22         08/02/22         1	06/27/22	07/01/22	3	16	2.5	1	(318.0)	(322.0)	1	0	0	-318	-318	8846	8614
07/11/22         07/15/22         2         10         1.75         1.5         0.0         0.0         0         0         0         0         0         0         0         0         9646         9410           07/18/22         07/22/22         1         20         0.25         1.5         888.0         884.0         1         888         1         0         0         10534         10294           07/25/22         07/29/22         1         20         0.25         1.5         1693.0         1689.0         1         1693         1         0         0         12227         11983           08/01/22         08/05/22         1         16         0.75         2         883.0         879.0         1         883         1         0         0         13110         12862           08/08/22         08/12/22         2         6         0.25         1.75         (920.0)         (924.0)         1         0         0         13318         13066           08/15/22         08/26/22         1         12         0.75         1.5         0.0         0.0         0         0         0         12398         12142           0	07/04/22	07/08/22	2	7	1	0.75	800.0	796.0	1	800	1	0	0	9646	9410
07/18/22       07/22/22       1       20       0.25       1.5       888.0       884.0       1       888       1       0       0       10534       10294         07/25/22       07/29/22       1       20       0.25       1.5       1693.0       1689.0       1       1693       1       0       0       12227       11983         08/01/22       08/05/22       1       16       0.75       2       883.0       879.0       1       883       1       0       0       13110       12862         08/08/22       08/12/22       2       6       0.25       2       208.0       204.0       1       208       1       0       0       13110       12862         08/05/22       08/19/22       3       9       0.25       1.75       (920.0)       (924.0)       1       0       0       12398       12142         08/21/22       08/26/22       1       12       0.75       1.5       0.0       0.0       0       0       0       12398       12142         08/29/22       09/02/22       1       5       0.25       1035.0       1031.0       1       1093       1       0	07/11/22	07/15/22	2	10	1.75	1.5	0.0	0.0	0	0	0	0	0	9646	9410
07/25/2207/29/221200.251.51693.01689.011693100122271198308/01/2208/05/221160.752883.0879.01883100131101286208/08/2208/12/22260.252208.0204.01208100131101286208/08/2208/12/22260.251.75(920.0)(924.0)100-920-920123981214208/22/2208/26/221120.751.50.00.00000134911323108/29/2209/02/22150.2511093.01089.011093100145261426209/12/2209/09/223190.250.251035.01031.011035100145261426209/12/2209/16/223180.750.5(1248.0)(1252.0)1100145761430409/26/2209/30/2211210.251298.01294.011298100166201634009/26/2209/30/221160.50.251055.01051.011055100166201634009/26/2209/30/221160.50	07/18/22	07/22/22	1	20	0.25	1.5	888.0	884.0	1	888	1	0	0	10534	10294
08/01/22         08/05/22         1         16         0.75         2         883.0         879.0         1         883         1         0         0         13110         12862           08/08/22         08/12/22         2         6         0.25         2         208.0         204.0         1         208         1         0         0         13110         12862           08/08/22         08/12/22         2         6         0.25         1.75         (920.0)         (924.0)         1         0         0         -920         -920         12398         12142           08/22/22         08/26/22         1         12         0.75         1.5         0.0         0.0         0         0         0         13149         12398           08/29/22         09/02/22         1         5         0.25         1         1093.0         1089.0         1         1093         1         0         0         13491         13231           09/05/22         09/09/22         3         18         0.75         0.5         (1248.0)         (1252.0)         1         1         0         0         14576         14304           09/26/22         0	07/25/22	07/29/22	1	20	0.25	1.5	1693.0	1689.0	1	1693	1	0	0	12227	11983
08/08/22         08/12/22         2         6         0.25         2         208.0         204.0         1         208         1         0         0         13318         13066           08/15/22         08/19/22         3         9         0.25         1.75         (920.0)         (924.0)         1         0         0         -920         -920         12398         12142           08/22/22         08/26/22         1         12         0.75         1.5         0.0         0.0         0         0         0         12398         12142           08/29/22         09/02/22         1         5         0.25         1         1093.0         1089.0         1         1093         1         0         0         13491         13231           09/05/22         09/09/22         3         19         0.25         0.25         1035.0         1031.0         1         1035         1         0         0         14526         14262           09/12/22         09/23/22         1         12         1         0.25         1298.0         1294.0         1         1298         1         0         0         14576         14304           09/26	08/01/22	08/05/22	1	16	0.75	2	883.0	879.0	1	883	1	0	0	13110	12862
08/15/22         08/19/22         3         9         0.25         1.75         (920.0)         (924.0)         1         0         0         -920         -920         12398         12142           08/22/22         08/26/22         1         12         0.75         1.5         0.0         0.0         0         0         0         0         12398         12142           08/29/22         09/02/22         1         5         0.25         1         1093.0         1089.0         1         1093         1         0         0         13491         13231           09/05/22         09/09/22         3         19         0.25         0.25         1035.0         1031.0         1         1035         1         0         0         14526         14262           09/12/22         09/16/22         3         18         0.75         0.5         (1248.0)         (1252.0)         1         0         0         14526         14262           09/19/22         09/23/22         1         12         1         0.25         1298.0         1294.0         1         1298         1         0         0         14576         14304           09/26/22	08/08/22	08/12/22	2	6	0.25	2	208.0	204.0	1	208	1	0	0	13318	13066
08/22/22         08/26/22         1         12         0.75         1.5         0.0         0.0         0         0         0         0         0         12398         12142           08/29/22         09/02/22         1         5         0.25         1         1093.0         1089.0         1         1093         1         0         0         13491         13231           09/05/22         09/09/22         3         19         0.25         0.25         1035.0         1031.0         1         1093         1         0         0         14526         14262           09/12/22         09/16/22         3         18         0.75         0.5         (1248.0)         (1252.0)         1         0         0         14526         14262           09/12/22         09/23/22         1         12         1         0.25         1298.0         1294.0         1         1298         1         0         0         14576         14304           09/26/22         09/30/22         1         16         0.5         0.25         1055.0         1051.0         1         1055         1         0         0         16620         16301           10/	08/15/22	08/19/22	3	9	0.25	1.75	(920.0)	(924.0)	1	0	0	-920	-920	12398	12142
08/29/22         09/02/22         1         5         0.25         1         1093.0         1089.0         1         1093         1         0         0         13491         13231           09/05/22         09/09/22         3         19         0.25         0.25         1035.0         1031.0         1         1035         1         0         0         14526         14262           09/12/22         09/16/22         3         18         0.75         0.5         (1248.0)         (1252.0)         1         0         0         -1248         -1248         13278         13010           09/19/22         09/23/22         1         12         1         0.25         1298.0         1294.0         1         1298         1         0         0         14576         14304           09/26/22         09/30/22         1         16         0.5         0.25         1055.0         1051.0         1         1055         1         0         0         16620         16340           10/03/22         10/07/22         1         12         0.25         0.25         266.0         262.0         1         266         1         0         0         16826	08/22/22	08/26/22	1	12	0.75	1.5	0.0	0.0	0	0	0	0	0	12398	12142
09/05/22         09/09/22         3         19         0.25         0.25         1035.0         1031.0         1         1035         1         0         0         14526         14262           09/12/22         09/16/22         3         18         0.75         0.5         (1248.0)         (1252.0)         1         0         0         -1248         -1248         13278         13010           09/19/22         09/23/22         1         12         1         0.25         1298.0         1294.0         1         1298         1         0         0         14576         14304           09/26/22         09/30/22         1         16         0.5         0.25         1055.0         1051.0         1         1055         1         0         0         14576         14304           09/26/22         09/30/22         1         12         0.25         0.25         1055.0         1051.0         1         1055         1         0         0         16620         16340           10/10/22         10/07/22         1         7         1.25         0.25         266.0         262.0         1         266         1         0         0         16886	08/29/22	09/02/22	1	5	0.25	1	1093.0	1089.0	1	1093	1	0	0	13491	13231
09/12/22         09/16/22         3         18         0.75         0.5         (1248.0)         (1252.0)         1         0         0         -1248         -1248         13278         13010           09/19/22         09/23/22         1         12         1         0.25         1298.0         1294.0         1         1298         1         0         0         14576         14304           09/26/22         09/30/22         1         16         0.5         0.25         1055.0         1051.0         1         1055         1         0         0         14576         14304           09/26/22         09/30/22         1         16         0.5         0.25         1055.0         1051.0         1         1055         1         0         0         15631         15355           10/03/22         10/07/22         1         7         1.25         0.25         266.0         262.0         1         266         1         0         0         16820         16602           10/17/22         10/21/22         2         10         2         1.75         0.0         0.0         0         0         0         16826         16602 <t< td=""><td>09/05/22</td><td>09/09/22</td><td>3</td><td>19</td><td>0.25</td><td>0.25</td><td>1035.0</td><td>1031.0</td><td>1</td><td>1035</td><td>1</td><td>0</td><td>0</td><td>14526</td><td>14262</td></t<>	09/05/22	09/09/22	3	19	0.25	0.25	1035.0	1031.0	1	1035	1	0	0	14526	14262
09/19/22         09/23/22         1         12         1         0.25         1298.0         1294.0         1         1298         1         0         0         14576         14304           09/26/22         09/30/22         1         16         0.5         0.25         1055.0         1051.0         1         1055         1         0         0         14576         14304           09/26/22         09/30/22         1         16         0.5         0.25         1055.0         1051.0         1         1055         1         0         0         15631         15355           10/03/22         10/07/22         1         12         0.25         0.25         989.0         985.0         1         989         1         0         0         16620         16340           10/10/22         10/14/22         1         7         1.25         0.25         266.0         262.0         1         266         1         0         0         16886         16602           10/17/22         10/21/22         2         10         2         1.75         0.0         0.0         0         0         0         16886         16602           10/24/22<	09/12/22	09/16/22	3	18	0.75	0.5	(1248.0)	(1252.0)	1	0	0	-1248	-1248	13278	13010
09/26/22         09/30/22         1         16         0.5         0.25         1055.0         1051.0         1         1055         1         0         0         15631         15355           10/03/22         10/07/22         1         12         0.25         0.25         989.0         985.0         1         989         1         0         0         16620         16340           10/10/22         10/14/22         1         7         1.25         0.25         266.0         262.0         1         266         1         0         0         16826         16602           10/17/22         10/21/22         2         10         2         1.75         0.0         0.0         0         0         0         16886         16602           10/24/22         10/28/22         3         9         2.5         0.25         (899.0)         (903.0)         1         0         0         -899         -899         15987         15699           10/31/22         11/04/22         1         7         1         1.75         0.0         0.0         0         0         0         15987         15699           11/07/22         11/04/22         1	09/19/22	09/23/22	1	12	1	0.25	1298.0	1294.0	1	1298	1	0	0	14576	14304
10/03/22         10/07/22         1         12         0.25         0.25         989.0         985.0         1         989         1         0         0         16620         16340           10/10/22         10/14/22         1         7         1.25         0.25         266.0         262.0         1         266         1         0         0         16820         16340           10/17/22         10/21/22         2         10         2         1.75         0.0         0.0         0         0         0         16826         16602           10/24/22         10/28/22         3         9         2.5         0.25         (899.0)         (903.0)         1         0         0         -899         -899         15987         15699           10/31/22         11/04/22         1         7         1         1.75         0.0         0.0         0         0         0         1         15987         15699           11/07/22         11/11/22         2         8         1.5         0.5         540.0         536.0         1         540         1         0         0         16527         16235	09/26/22	09/30/22	1	16	0.5	0.25	1055.0	1051.0	1	1055	1	0	0	15631	15355
10/10/22         10/14/22         1         7         1.25         0.25         266.0         262.0         1         266         1         0         0         16886         16602           10/17/22         10/21/22         2         10         2         1.75         0.0         0.0         0         0         0         0         16886         16602           10/24/22         10/28/22         3         9         2.5         0.25         (899.0)         (903.0)         1         0         0         -899         -899         15987         15699           10/31/22         11/04/22         1         7         1         1.75         0.0         0.0         0         0         0         1         5987         15699           11/07/22         11/11/22         2         8         1.5         0.5         540.0         536.0         1         540         1         0         0         16527         16235	10/03/22	10/07/22	1	12	0.25	0.25	989.0	985.0	1	989	1	0	0	16620	16340
10/17/22         10/21/22         2         10         2         1.75         0.0         0.0         0         0         0         0         16886         16602           10/24/22         10/28/22         3         9         2.5         0.25         (899.0)         (903.0)         1         0         0         -899         -899         15987         15699           10/31/22         11/04/22         1         7         1         1.75         0.0         0.0         0         0         0         15987         15699           11/07/22         11/11/22         2         8         1.5         0.5         540.0         536.0         1         540         1         0         0         16527         16235	10/10/22	10/14/22	1	7	1.25	0.25	266.0	262.0	1	266	1	0	0	16886	16602
10/24/22         10/28/22         3         9         2.5         0.25         (899.0)         (903.0)         1         0         0         -899         -899         15987         15699           10/31/22         11/04/22         1         7         1         1.75         0.0         0.0         0         0         0         0         15987         15699           11/07/22         11/11/22         2         8         1.5         0.5         540.0         536.0         1         540         1         0         0         16527         16235	10/17/22	10/21/22	2	10	2	1.75	0.0	0.0	0	0	0	0	0	16886	16602
10/31/22         11/04/22         1         7         1         1.75         0.0         0.0         0         0         0         0         15987         15699           11/07/22         11/11/22         2         8         1.5         0.5         540.0         536.0         1         540         1         0         0         16527         16235	10/24/22	10/28/22	3	9	2.5	0.25	(899.0)	(903.0)	1	0	0	-899	-899	15987	15699
11/07/22         11/11/22         2         8         1.5         0.5         540.0         536.0         1         540         1         0         0         16527         16235	10/31/22	11/04/22	1	7	1	1.75	0.0	0.0	0	0	0	0	0	15987	15699
	11/07/22	11/11/22	2	8	1.5	0.5	540.0	536.0	1	540	1	0	0	16527	16235
<u>11/14/22</u> <u>11/18/22</u> <u>3</u> <u>7</u> <u>1.75</u> <u>3</u> <u>0.0</u> <u>0.0</u> <u>0</u> <u>0</u> <u>0</u> <u>0</u> <u>16527</u> <u>16235</u>	11/14/22	11/18/22	3	7	1.75	3	0.0	0.0	0	0	0	0	0	16527	16235

OOS Dates		pw	Ν	vup	vdn	osnp	NOnp\$4	ont	ownp	ownt	ollt	odd	EQ	NetEq
11/21/22	11/25/22	3	9	1	1.75	0.0	0.0	0	0	0	0	0	16527	16235
11/28/22	12/02/22	2	8	1	2.5	0.0	0.0	0	0	0	0	0	16527	16235
12/05/22	12/09/22	2	7	1.5	2.5	0.0	0.0	0	0	0	0	0	16527	16235
12/12/22	12/16/22	1	16	0.75	2	0.0	0.0	0	0	0	0	0	16527	16235
12/19/22	12/23/22	1	16	0.75	2	0.0	0.0	0	0	0	0	0	16527	16235
12/26/22	12/30/22	3	13	0.5	2.5	0.0	0.0	0	0	0	0	0	16527	16235
01/02/23	01/06/23	1	10	0.75	0.5	0.0	0.0	0	0	0	0	0	16527	16235
01/09/23	01/13/23	1	10	0.75	0.5	285.0	281.0	1	285	1	0	0	16812	16516
01/16/23	01/20/23	3	20	0.25	0.75	(43.0)	(47.0)	1	0	0	-43	-43	16769	16469
01/23/23	01/27/23	1	5	0.25	1	895.0	891.0	1	895	1	0	0	17664	17360
01/30/23	02/03/23	3	9	0.25	1	1177.0	1173.0	1	1177	1	0	0	18841	18533
02/06/23	02/10/23	1	17	1	0.75	(376.0)	(380.0)	1	0	0	-376	-376	18465	18153
02/13/23	02/17/23	3	9	0.25	0.75	(173.0)	(177.0)	1	0	0	-173	-173	18292	17976
02/20/23	02/24/23	3	9	0.25	0.75	295.0	287.0	2	295	2	0	0	18587	18263
02/27/23	03/03/23	2	13	0.75	1	0.0	0.0	0	0	0	0	0	18587	18263
03/06/23	03/10/23	1	5	2	0.25	428.0	424.0	1	428	1	0	0	19015	18687
03/13/23	03/17/23	3	11	0.75	3	736.0	732.0	1	736	1	0	0	19751	19419
03/20/23	03/24/23	3	9	1.25	0.25	(295.0)	(299.0)	1	0	0	-295	-295	19456	19120
03/27/23	03/31/23	3	6	0.75	1.25	569.0	565.0	1	569	1	0	0	20025	19685
04/03/23	04/07/23	2	5	1.25	0.75	(381.0)	(385.0)	1	0	0	-381	-381	19644	19300
04/10/23	04/14/23	2	7	0.25	1	144.0	132.0	3	144	2	0	0	19788	19432
04/17/23	04/21/23	1	16	1	0.25	0.0	0.0	0	0	0	0	0	19788	19432
04/24/23	04/28/23	1	16	1	0.25	0.0	0.0	0	0	0	0	0	19788	19432
05/01/23	05/05/23	1	5	0.5	2.25	77.0	73.0	1	77	1	0	0	19865	19505
05/08/23	05/12/23	3	8	0.5	1.5	303.0	299.0	1	303	1	0	0	20168	19804
05/15/23	05/19/23	2	17	0.25	0.25	1045.0	1041.0	1	1045	1	0	0	21213	20845
05/22/23	05/26/23	1	9	0.25	0.5	1251.0	1247.0	1	1251	1	0	0	22464	22092
05/29/23	06/02/23	2	10	0.75	2	628.0	624.0	1	628	1	0	0	23092	22716
06/05/23	06/09/23	3	19	0.25	0.25	(1054.0)	(1062.0)	2	0	0	-538	-1054	22038	21654
06/12/23	06/16/23	1	5	0.25	0.5	464.0	460.0	1	464	1	0	0	22502	22114
06/19/23	06/23/23	2	8	0.25	1	(191.0)	(195.0)	1	0	0	-191	-191	22311	21919
06/26/23	06/30/23	3	14	0.5	1.25	19.0	11.0	2	518	1	-499	-499	22330	21930
07/03/23	07/07/23	3	9	0.5	0.75	(48.0)	(56.0)	2	99	1	-147	-147	22282	21874
07/10/23	07/14/23	3	9	0.5	2.5	302.0	298.0	1	302	1	0	0	22584	22172
07/17/23	07/21/23	3	8	0.25	1	355.0	351.0	1	355	1	0	0	22939	22523
07/24/23	07/28/23	3	8	0.5	0.25	28.0	20.0	2	302	1	-274	-274	22967	22543
07/31/23	08/04/23	3	5	0.5	2.5	(371.0)	(375.0)	1	0	0	-371	-371	22596	22168
08/07/23	08/11/23	1	14	0.25	0.75	0.0	0.0	0	0	0	0	0	22596	22168
08/14/23	08/18/23	2	15	1.25	0.25	1123.0	1119.0	1	1123	1	0	0	23719	23287
08/21/23	08/25/23	2	15	1.25	0.25	253.0	249.0	1	253	1	0	0	23972	23536
08/28/23	09/01/23	2	17	0.25	0.25	1103.0	1099.0	1	1103	1	0	0	25075	24635
09/04/23	09/08/23	1	13	0.5	0.75	(470.0)	(474.0)	1	0	0	-470	-470	24605	24161
09/11/23	09/15/23	2	17	0.25	0.75	0.0	0.0	0	0	0	0	0	24605	24161
09/18/23	09/22/23	1	8	0.25	0.25	1304.0	1300.0	1	1304	1	0	0	25909	25461
09/25/23	09/29/23	2	16	0.25	0.75	(128.0)	(132.0)	1	0	0	-128	-128	25781	25329
10/02/23	10/06/23	2	8	1.5	0.5	0.0	0.0	0	0	0	0	0	25781	25329
10/09/23	10/13/23	3	9	0.25	0.75	896.0	888.0	2	896	2	0	0	26677	26217
10/16/23	10/20/23	3	8	1.25	0.25	1021.0	1017.0	1	1021	1	0	0	27698	27234
10/23/23	10/27/23	2	6	0.25	1.25	(915.0)	(923.0)	2	112	1	-1027	-1027	26783	26311
10/30/23	11/03/23	3	13	1	0.25	617.0	613.0	1	617	1	0	0	27400	26924
11/06/23	11/10/23	3	13	0.75	0.25	(138.0)	(146.0)	2	337	1	-475	-475	27262	26778
11/13/23	11/17/23	2	7	2.5	0.25	0.0	0.0	0	0	0	0	0	27262	26778
11/20/23	11/24/23	1	9	0.5	0.25	50.0	46.0	1	50	1	0	0	27312	26824
11/27/23	12/01/23	3	9	0.5	0.5	0.0	0.0	0	0	0	0	0	27312	26824
12/04/23	12/08/23	3	18	2	0.25	0.0	0.0	0	0	0	0	0	27312	26824
12/11/23	12/15/23	3	15	0.75	0.75	978.0	974.0	1	978	1	0	0	28290	27798
12/18/23	12/22/23	3	5	0.25	1.25	(304.0)	(316.0)	3	0	0	-194	-304	27986	27482
12/25/23	12/29/23	3	5	0.25	1.75	(315.0)	(319.0)	1	0	0	-315	-315	27671	27163
01/01/24	01/05/24	2	7	0.5	1.75	(31.0)	(35.0)	1	0	0	-31	-31	27640	27128

OOS Dates		pw	N	vup	vdn	osnp	NOnp\$4	ont	ownp	ownt	ollt	odd	EQ	NetEq
01/08/24	01/12/24	2	7	0.5	2	765.0	761.0	1	765	1	0	0	28405	27889
01/15/24	01/19/24	1	5	0.5	0.25	0.0	0.0	0	0	0	0	0	28405	27889
01/22/24	01/26/24	3	9	0.75	0.75	264.0	256.0	2	264	2	0	0	28669	28145
01/29/24	02/02/24	3	16	0.5	1.5	0.0	0.0	0	0	0	0	0	28669	28145
02/05/24	02/09/24	3	5	0.25	3.25	494.0	490.0	1	494	1	0	0	29163	28635
02/12/24	02/16/24	1	18	0.75	0.25	329.0	325.0	1	329	1	0	0	29492	28960
02/19/24	02/23/24	1	18	1	0.5	0.0	0.0	0	0	0	0	0	29492	28960
02/26/24	03/01/24	1	18	1	0.5	0.0	0.0	0	0	0	0	0	29492	28960
03/04/24	03/08/24	3	13	1	1	0.0	0.0	0	0	0	0	0	29492	28960
03/11/24	03/15/24	2	12	0.25	0.5	0.0	0.0	0	0	0	0	0	29492	28960
03/18/24	03/22/24	3	6	3.25	2	0.0	0.0	0	0	0	0	0	29492	28960
03/25/24	03/29/24	3	13	0.75	0.25	77.0	73.0	1	77	1	0	0	29569	29033
04/01/24	04/05/24	2	5	2	1	(369.0)	(373.0)	1	0	0	-369	-369	29200	28660
04/08/24	04/12/24	2	6	0.5	0.25	(469.0)	(473.0)	1	0	0	-469	-469	28731	28187
04/15/24	04/19/24	2	5	0.75	0.75	1625.0	1621.0	1	1625	1	0	0	30356	29808
04/22/24	04/26/24	1	7	1.25	0.5	(1023.0)	(1027.0)	1	0	0	-1023	-1023	29333	28781
04/29/24	05/03/24	2	6	2	1	(1018.0)	(1022.0)	1	0	0	-1018	-1018	28315	27759
05/06/24	05/10/24	1	6	1	1	136.0	132.0	1	136	1	0	0	28451	27891
05/13/24	05/17/24	2	12	1.25	0.25	0.0	0.0	0	0	0	0	0	28451	27891
05/20/24	05/24/24	3	7	0.25	2	189.0	185.0	1	189	1	0	0	28640	28076
05/27/24	05/31/24	2	14	0.75	1	0.0	0.0	0	0	0	0	0	28640	28076
06/03/24	06/07/24	2	5	0.25	2.5	1009.0	1005.0	1	1009	1	0	0	29649	29081
06/10/24	06/14/24	3	7	0.5	2.5	720.0	716.0	1	720	1	0	0	30369	29797
06/17/24	06/21/24	3	7	0.5	2.5	0.0	0.0	0	0	0	0	0	30369	29797
06/24/24	06/28/24	2	10	1.25	1	(304.0)	(308.0)	1	0	0	-304	-304	30065	29489
07/01/24	07/05/24	2	10	1.25	2	0.0	0.0	0	0	0	0	0	30065	29489
07/08/24	07/12/24	2	10	1.25	2	0.0	0.0	0	0	0	0	0	30065	29489
07/15/24	07/19/24	3	5	1.75	0.25	370.0	362.0	2	1304	1	-934	-934	30435	29851
07/22/24	07/26/24	2	12	1.75	0.75	1844.0	1840.0	1	1844	1	0	0	32279	31691
07/29/24	08/02/24	1	10	1.5	1.5	1710.0	1706.0	1	1710	1	0	0	33989	33397
08/05/24	08/09/24	1	10	1.5	1.5	(1318.0)	(1322.0)	1	0	0	-1318	-1318	32671	32075
08/12/24	08/16/24	1	5	1.75	0.25	1152.0	1148.0	1	1152	1	0	0	33823	33223
08/19/24	08/23/24	2	20	2	1.5	(35.0)	(39.0)	1	0	0	-35	-35	33788	33184
08/26/24	08/30/24	1	14	3	2.25	0.0	0.0	0	0	0	0	0	33788	33184
09/02/24	09/06/24	3	5	0.5	3	(1330.0)	(1338.0)	2	0	0	-1164	-1330	32458	31846
09/09/24	09/13/24	3	9	1	0.5	1543.0	1539.0	1	1543	1	0	0	34001	33385
09/16/24	09/20/24	3	6	1.25	2.5	(5.0)	(9.0)	1	0	0	-5	-5	33996	33376
09/23/24	09/27/24	3	11	0.25	0.5	229.0	225.0	1	229	1	0	0	34225	33601
09/30/24	10/04/24	3	10	1.25	0.75	(758.0)	(762.0)	1	0	0	-758	-758	33467	32839
10/07/24	10/11/24	3	20	2	0.75	0.0	0.0	0	0	0	0	0	33467	32839
10/14/24	10/18/24	1	5	1.5	0.5	0.0	0.0	0	0	0	0	0	33467	32839
10/21/24	10/25/24	1	17	0.5	0.5	259.0	255.0	1	259	1	0	0	33726	33094
10/28/24	11/01/24	1	15	0.75	1	0.0	0.0	0	0	0	0	0	33726	33094
11/04/24	11/08/24	3	11	0.25	2	1358.0	1354.0	1	1358	1	0	0	35084	34448
11/11/24	11/15/24	3	8	0.75	3.5	0.0	0.0	0	0	0	0	0	35084	34448
11/18/24	11/22/24	3	10	0.75	2	263.0	259.0	1	263	1	0	0	35347	34707
11/25/24	11/29/24	3	10	0.75	2	0.0	0.0	0	0	0	0	0	35347	34707
12/02/24	12/06/24	3	10	0.25	0.5	1253.0	1249.0	1	1253	1	0	0	36600	35956
12/09/24	12/13/24	1	5	2.25	1	0.0	0.0	0	0	0	0	0	36600	35956
12/16/24	12/20/24	1	6	0.25	0.25	(1264.0)	(1268.0)	1	0	0	-1264	-1264	35336	34688
12/23/24	12/27/24	1	10	0.5	1.5	0.0	0.0	0	0	0	0	0	35336	34688
12/30/24	01/03/25	2	17	1	1.25	0.0	0.0	0	0	0	0	0	35336	34688

# Appendix I: n<sup>th</sup> Order Polynomial Next Bar's Forecast Math

### What is the n<sup>th</sup> Order Polynomial?

The n<sup>th</sup> Order Polynomial, also called the n<sup>th</sup> Order Fixed Memory Polynomial, is simply the least square fit of a polynomial of the form  $b_0+b_1*t+b_2*t^2+b_3*t^3+...b_n*t^n$  to a *fixed* number of past data points. Where t is discrete time bars. Time could be daily bars or 5-minute bars. We use the term "Fixed Memory" to designate that only a fixed number of data points are used to calculate the polynomial coefficients. It is assumed that the time bars occur at fixed intervals of time so tic bars would not be appropriate for this analysis. Least squares are a mathematical technique where the squared vertical distance between the data and the curve that is being fit to the data is minimized. When the net squared distance (also called the sum of the squared errors) is minimized, a unique set of coefficients  $b_0, b_1, b_2, ..., b_n$  for the polynomial is determined. This type of error minimization is mathematically solvable and is widely used in science and mathematics.

For a 4<sup>th</sup> order polynomial equation, the least squares coefficients are obtained from the solution of the following matrix equation.

T $\sum t \sum t^2 \sum t^3 \sum t^4$	a₀	∑p(t)
$\sum t \sum t^2 \sum t^3 \sum t^4 \sum t^5$	b <sub>0</sub>	∑(p(t)*t)
$\sum t^2 \sum t^3 \sum t^4 \sum t^5 \sum t^6$	<b>c</b> 0 =	∑(p(t)*t²)
$\sum t^3 \sum t^4 \sum t^5 \sum t^6 \sum t^7$	d₀	∑(p(t)*t³)
$\sum t^4 \sum t^5 \sum t^6 \sum t^7 \sum t^8$	e <sub>0</sub>	∑(p(t)*t⁴)

where

**p(T)** is the current bar's price, **p(T-1)** is the previous bar's price and **p(1)** is the price **T** bars ago. **T** is the number of Bars in the Least Squares estimation  $\sum p(t)$  is the summation of prices from **t=1 to T** bars  $\sum p(t)^*t$  is the summation of prices times t from **t=1 to T** bars  $\sum t$  is the summation of the integer t from **t=1 to T** bars  $\sum t^2$  is the summation of the integer t squared from **t=1 to T** bars etc.

Once the coefficients to the polynomial have been solved for, we generate the forecast for the next bar's price which is given for the equation by:

### $P_f = a_0 + b_0^*(T+1) + c_0^*(T+1)^2 + d_0^*(T+1)^3 + e_0^*(T+1)^4$

Where  $\mathbf{P}_{\mathbf{f}}$  stands for price forecast.

With these coefficients, we can also generate the forecast for the next bar's *velocity* and *velocity* by the equations:

### Velocity(T+1) = $dP_f/dt = b_0 + 2c_0^*(T+1) + 3d_0^*(T+1)^2 + 4e_0^*(T+1)^3$

### Acceleration(t+1) = $d^2P_f/d^2t = 2 c_0 + 6d_0*(T+1) + 12e_0*(T+1)^2$

We use the next bar forecast because changes in the trend are more quickly reflected in the forecast price, velocity and velocity than in the end point price, velocity and velocity.

Programs that solve for the solution to matrix equations can be found in the book "Numerical Recipes" by W. Press, et. al. However this type of matrix equation solvers is very slow and for these types of problems are unstable. They cause numerical errors and floating-point overflows due to matrix inversion ill conditioning which produces false results.

# Appendix I: n<sup>th</sup> Order Polynomial Next Bar's Forecast Math

Fortunately, these types of problems can be solved by a fast, efficient and accurate algorithm using Discrete Orthogonal Legendre Polynomials. This method is explained in detail in Norman Morrison' book entitled "Introduction to Sequential Smoothing and Prediction", Chapter 7 page 223., referenced at the end of this section.

Without going into detail here (see Morrison reference), the polynomial filter can now be represented by:

$$P_{e}(t) = \sum_{j=0}^{n} \beta_{j} * \phi_{j}(t) = 0 \text{ to n}$$

Where **n** is the polynomial order, **T** is the total number of Bars of data used in the Least Squares fit and

$$\beta_{j=} \sum_{k=0}^{T-1} p(t-T+k)*\phi_j(k)$$

### $\phi_i(t)$ = the normalized discrete Legendre polynomial. t = an integer from 0 to T

The coefficients,  $\beta_0$ ,  $\beta_1$ ,  $\beta_2$ ,  $\beta_3$ , , , , $\beta_n$  for a **n**<sup>th</sup> order polynomial can now be solved for by the equation above and we can generate the forecast for the next bar's close, velocity and velocity which are given by the equations

 $P_{F}(T+1) = \beta_{0}*\phi_{0}(T+1) + \beta_{1}*\phi_{1}(T+1) + \beta_{2}*\phi_{2}(T+1) + \beta_{3}*\phi_{3}(T+1) + ... + \beta_{n}*\phi_{n}(T+1)$ 

 $Velocity = (dP_{F}/dt)_{(T+1)} = \beta_{1}^{*}(d\phi_{1}/dt)_{(T+1)} + \beta_{2}^{*}(d\phi_{2}/dt)_{(T+1)} + \beta_{3}^{*}(d\phi_{3}/dt)_{(T+1)} + + \beta_{n}^{*}(d\phi_{n}/dt)_{(T+1)} + \beta_{n}^{*}(d\phi_{n}/dt)_{(T+$ 

Acceleration =  $(d^2 P_F/d^2 t)_{(T+1)} = \beta_2 * (d^2 \phi_2/d^2 t)_{(T+1)} + \beta_3 * (d^2 \phi_3/d^2 t)_{(T+1)} + ... + \beta_n * (d^2 \phi_n/d^2 t)_{(T+1)}$ 

### The n<sup>th</sup> Order Fixed Memory Forecast Next Bar's Velocity Strategy Defined

The least squares forecast is constructed by solving for the least squares coefficients  $\beta_1$ ,  $\beta_2$ , , ,  $\beta_n$  at each bar using the last **T** bars of closing prices and the Discrete Orthogonal Legendre Polynomial equations for  $\beta_j$  above. Then **Velocity** =  $d^2P_F(T+1)/d^2t$  is constructed from the velocity equation above and plotted under the price chart. In general, what we will be doing is following the plotted curve of **Velocity** which is calculated at each bar from the previous T bars. When the velocity is greater than a threshold amount *vup* we will go long. When the velocity is less than a threshold amount *-vdn* we will go short.

### Buy Rule:

**IF Velocity** is greater than the threshold amount *vup* then buy at the market.

Sell Rule:

IF Velocity is less than the threshold amount -vdn then sell at the market.

### **References**

1. Morrison, Norman "Introduction to Sequential Smoothing and Prediction", McGraw-Hill Book Company, New York, 1969.

# The Normalization Multiplier

### What is the Multiplier?

The n<sup>th</sup> Order Fixed Memory Polynomial, also called an n<sup>th</sup> Order Polynomial, is the least square fit of a polynomial of the form  $b_0+b_1*t+b_2*t^2+b_3*t^3+...+b_n*t^n$  to a *fixed* number of past data points. Where **t** is discrete time bars. Time could be daily bars or 5-minute bars. We use the term "Fixed Memory" to designate that only a fixed number of data points are used to calculate the polynomial coefficients. It is assumed that the time bars occur at fixed intervals of time so tic bars would not be appropriate for this analysis. Least squares are a mathematical technique where the squared vertical distance between the data and the curve that is being fit to the data is minimized. When the net squared distance (also called the sum of the squared errors) is minimized, a unique set of coefficients  $b_0, b_1, b_2, ..., b_n$  for the polynomial is determined. This type of error minimization is mathematically solvable and is widely used in science and mathematics. Once the  $b_n$  coefficients are found using a lookback period of **T** bars to calculate the  $b_n$  coefficients, then the next bar's estimate (**T+1**) of the n<sup>th</sup> order polynomial velocity and acceleration can be easily found by the equations below.

### $Velocity(T+1) = dP_{f} / dt = b_{0} + 2c_{0}^{*}(T+1) + 3d_{0}^{*}(T+1)^{2} + 4e_{0}^{*}(T+1)^{3} + \dots + n^{*}b_{n}^{*}(T+1)^{n-1}$

### Acceleration(t+1) = $d^2P_f/d^2t = 2 c_0 + 9d_0^*(T+1) + 12e_0^*(T+1)^3 + ..... + n^*(n-1)^*b_n^*(T+1)^{n-2}$

Please see the *n*<sup>th</sup> Order Fixed Memory Polynomial Next Bar's Forecast Math section for a more detailed explanation.

For any tradable, the inputs to the polynomial are the **polynomial degree (Order)** and the number or lookback bars **N** (denoted by T in equations above). When we plot the velocity or acceleration, we notice that the amplitude, and the maximum and minimum values of the velocity or acceleration vary quite significantly with different degree and N inputs.

Below is a table of the standard deviation (SD) of the 56340 calculated Velocity values for different **degree** and **N** inputs. We used 1min bars of the E-Mini from 8/1/2014 to 2/20/2015 to generate this table. As one can see the standard deviation of the velocity for each degree and N vary greatly. For instance, for degree=4, N=20 the SD is 6.8 times the SD for degree=1, N=20. This creates problems when trying to determine the correct ranges for vup/vdn and aup/adn during optimization.

### @ES.D 5 min bars Date Range 1140801 to 1150220 Total Number of Bars=56340 Norm=0 FixmVn Multiplier= 1/SD to Scale Velocity pw and N Range to One SD

Degree	Ν	SD	1/SD
1	20	0.1902	5.2565
1	30	0.1540	6.4916
1	40	0.1328	7.5279
1	50	0.1183	8.4502
1	60	0.1077	9.3320
1	70	0.0996	10.0440
avg		0.1338	7.8430
2	20	0.4351	2.2982
2	30	0.3443	2.9046
2	40	0.2936	3.4060
2	50	0.1583	3.8275
2	60	0.2371	4.2180
2	70	0.2173	4.6010

# The Normalization Multiplier

avg		0.2981	3.5425
3	20	0.7854	1.2732
3	30	0.5933	1.6855
3	40	0.4973	2.0111
3	50	0.4347	2.3005
3	60	0.3949	2.5324
3	70	0.3656	2.7352
avg		0.5119	2.0897
4	20	1.2924	0.7738
4	30	0.9279	1.0777
4	40	0.7582	1.3189
4	5	0.6542	1.5285
4	60	0.5804	1.7228
4	70	0.5314	1.8818
avg		0.7908	1.3839

The problem may get worse when we want to find good inputs for other tradables. Other tradables, because of their scales and tick size have much different Velocity ranges then the E-Mini for the same degree and N. Thus, the NS search ranges have to be different for each different tradable.

To solve this problem and to have a standard search space for each tradable, I created a **Mult** input for each FixmXVA Velocity and Acceleration strategy and indicator. If each tradable's Velocity is multiplied by a number such that the standard deviation of that tradable's Velocity is around one, then the search space for vup and vdn for each tradable would be 0 to 3.5 SDs and we wouldn't have to change the TS search space every time we wanted to examine a new stock or future. The complicated equations that I use to normalize the ranges to one standard deviation were derived using the software TableCurve 3D, automated surface and equation discovery.