4th Order Polynomial System on S&P500 Futures 5min Bars

In a previous article on the British Pound futures **(BP)**, it was shown how the application of a curve generated by a third degree polynomial could be used to develop a daily system to buy and sell the British Pound futures. Here we will use a variation of that previously developed system to create a system to trade the S&P500 futures on an intraday basis using 5 minute bar data from the S&P Sept/99 futures contract..

The 4th Order Polynomial System.

In the article, entitled "The British Pound Cubed", the least Squares technique was used to fit a cubic polynomial of the form $a_0+b_0*t+c_0*t^2+d_0*t^3$ through 30+ closing prices of the BP. This least square curve then served as a proxy for the market trend. When the curve moved up by a certain percent from its previous local low, the trend was assumed to have changed to the upside and a buy signal was given. When the curve moved down by a certain percent from its previous local high, the trend was assumed to have changed to the downside and a sell signal was given.

Due to the fast changing nature of intraday movements, here we will use the least squares technique to fit a 4th order polynomial curve of the form $a_0+b_0*t+c_0*t^2+d_0*t^3+e_0*t^4$ to create a curve that will serve as a better proxy for the intraday market trend of the S&P futures. The 4th order polynomial should respond faster to changes in the intraday trend and model the trend changes more accuracy than the cubic curve version .

What is Least Squares?

Least squares is a mathematical technique where at each bar of data, the squared distance between the data and the curve that is being fit to the data is minimized. When the net squared distance (also called the sum of the squared errors) is minimized, a unique set of coefficients a_0,b_0,c_0, d_0 and e_0 for the polynomial is determined.

For the 4th order polynomial equation, the least squares coefficients are obtained from the solution of the following matrix equation.

$$\begin{vmatrix} T & \Sigma t & \Sigma t^2 & \Sigma t^3 & \Sigma t^4 \\ \Sigma t & \Sigma t^2 & \Sigma t^3 & \Sigma t^4 & \Sigma t^5 \\ \Sigma t^2 & \Sigma t^3 & \Sigma t^4 & \Sigma t^5 & \Sigma t^6 \\ \Sigma t^3 & \Sigma t^4 & \Sigma t^5 & \Sigma t^6 & \Sigma t^7 \\ \Sigma t^4 & \Sigma t^5 & \Sigma t^6 & \Sigma t^7 & \Sigma t^8 \end{vmatrix} \begin{vmatrix} a_0 \\ b_0 \\ c_0 \\ d_0 \\ e_0 \end{vmatrix} = \begin{vmatrix} \Sigma p(t) \\ \Sigma (p(t)*t) \\ \Sigma (p(t)*t^2) \\ \Sigma (p(t)*t^3) \\ \Sigma (p(t)*t^4) \end{vmatrix}$$

where

p(T) is today's price, **p(T-1)** is yesterdays price and **p(1)** is the price **T** days ago.

T is the number of Bars in the Least Squares estimation

$\sum$**p(t)** is the summation of prices from **t=1 to T** days
$\sum$**p(t)*t** is the summation of prices times t from **t=1 to T** days
$\sum$**t** is the summation of the integer **t** from **t=1 to T** days
$\sum t^2$ is the summation of the integer **t** squared from **t=1 to T** days
etc.

Once the coefficients to the polynomial have been solved for we generate the forecast for the next day's price which is given for the equation by:

$$P_f = a_0 + b_0*(T+1) + c_0*(T+1)^2 + d_0*(T+1)^3 + e_0*(T+1)^4$$

Where P_f stands for price forecast.

We use the next day forecast price because changes in the trend are more quickly reflected in the forecast price than in the end point price.

The 4th Order Polynomial System Defined
The least squares forecast is constructed by solving for the least squares coefficients at each 5min Bar using the last **T** bars of SP 5minute close prices. P_f is then constructed from the equation above and plotted under the price chart. In general what we will be doing is following the plotted curve of P_f . When the curve increases by a percentage amount *pctup* from the previous prior low of the curve we will go long. When the curve falls by the percentage amount *pctdn* from the previous prior high of the curve we will go short.

Buy Rule:

- **IF P_f** has moved up by more than the percentage amount of *pctup* from the lowest low recorded in P_f while short then buy one S&P Sep/99 contract at the market.

Sell Rule:

- **IF P_f** has moved down by more than the percentage amount *pctdn* from the highest high recorded in P_f while long then sell one S&P Sep/99 contract at the market.

Exit Rule

- Exit all trades at close of each day with a market on close order.
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Walk Forward Optimization
Walk forward optimization will be used here because of the changing nature of the intraday S&P futures market. Intraday price dynamics are constantly changing due to current economic surprises, events and trader sentiment. Also the time of year changes the nature of intraday markets, such as the seasons, holidays, vacation time, etc. As such, optimizations on intraday data performed 3 months ago may no longer be representative of today's intraday price dynamics.

The walk forward procedure will be applied as follows. A period of 4 weeks from the start of the S&P Sept/99 5 minute bar data, July 7th, 1999 through August 5th, 1999, is chosen and system parameter values are found through optimization on this intraday data segment. The parameter

values found are then applied to the out-of-sample 5 minute intraday bar data following the test segment which in this case is August 6th, 1999 to August 13th, 1999.

Why a 4 week intraday data test segment? Why not 2 weeks or 3 months? Well the answer is that there is no correct ratio of test data needed to produce good one week intraday out-of-sample results. By experimenting with different window lengths, the four to one ratio seemed to work well. In walk forward testing, enough data is needed to model most of the price dynamics that will be encountered in the out-of-sample segment, but not so much data that when the price dynamics start to change they are swamped by the weight of distant past data price dynamics that no longer are valid. An important unspoken point in walk forward testing is that if you can not get good results in the out-of-sample segments, then the price dynamics cannot be modeled with the system . This means that real time performance will be random using the model. Traders observe this type of random performance (that is it looks great on paper but falls apart in real time) when trying systems based on curve fitting or anecdotal "proof" (looking at 3 or 4 successful cases only) without any out-of-sample testing .

Finding The System Parameters Using Walk Forward Optimization

There are four system parameters to find *T, pctup, and pctdn*. The best parameters will be defined as those values that give the best Net Profits with the minimum drawdown and minimum largest losing trades. In addition, the results should be stable, e.g. the profits, wins, and drawdowns should not change by much as the parameters move by a small amount away from their optimum values. Also in choosing the "best" parameters, only those parameters sets whose maximum consecutive losses were 4 or less were considered. Optimization is defined as the search for the parameter values that give the best results as defined above. It should be noted that in this stage of system development, the only thing indicated by the optimum values that are found in the test portion is that the data has been *curve fitted* as best it can with this system. Without further testing on out-of-sample data there is no way to tell if the system will work in the future.

It is not well known, but almost any real time series or even a random time series defined over a fixed number of bars can be curve fitted rather easily. The performance results and the statistical measurements that validate this performance of the curve fit will look excellent giving the false illusion of future profitability. However, the truth is that these excellent performance and associated statistics on the test section in no way validate how the system will perform on data it has not been optimized on. Only out-of-sample testing, that is testing on data the parameters were not derived on, can determine if the parameters found in the test section have captured the price dynamics. For instance in the Least square polynomial fit process the error minimization forces the generated curve to fit the past data like a glove. It's almost impossible not to get an excellent fit with excellent statistical results. Unfortunately, this excellent fit in no way implies that the system will perform equally well on out-of-sample data, it just tells us we have a very good curve fit.

Results

Figure 1 presents a table of the test window optimum parameters for the S&P Sep/99 5min data series.

Start Date	End Date	T	Pctup	Pctdn
7/7/99	8/5/99	60bars	0.90%	0.75%

Figure 1 Optimum Parameter Values For Test Data Segment

Figures 2 presents the performance summary using the optimum parameters for the test segment shown in Figure 1.

Figure 3 presents the performance summary of the out-of-sample data segment from 8/6/99 to 8/13/99. This performance represents what would have happened in *real time* if one used the parameters found in the test section. Slippage, and commissions are not included.

Figure 4 presents a TradeStation 2000i trade by trade summary from 8/6/99 to 8/13//99. Note only the out of sample trades are presented here for the in sample trades were generated by the curve fit.

Figures 5A through 5C present the 5 minute bar charts of the S&P Sep/99 futures with the 4^{th} order Polynomial Curve and all the buy and sell signals from the trade by trade summary of Figure 4 indicated on the charts.

Discussion of System Performance

As can be observed from the test sample Performance summary in Figure 2 and the out-of-sample performance summary of Figure 3, the out-of-sample performance was better than the test sample performance. This better performance in the out-of-sample section was due to chance but does indicate that 4 weeks of test data was enough to capture the intraday price dynamics of the S&P Sep/99 contract.

Observing the out-of-sample trade by trade summary of Figure 4, we can see that the system did equally well on longs and shorts. We won't be in a bull market forever, and the ability to trade on the short side will be of value. The average trade (win & loss) was $696 in the test section and $647 in the out-of-sample section indicating stability in the parameter selection. There were no really big winners or big losers indicating steady returns.

In observing the charts we can see that the system did very well in catching every major intraday trend of the S&P Sep/99 contract. As can be seen, many times the 4^{th} order polynomial curve overshoots the price curve because the curve is plotting the next bar's estimated value based upon the previous 60 bars. However, as we can observe from the charts, while this overshoot is disconcerting, it is not a problem for there is very little lag between the curve and the price bars at the price turning points. This no lag feature of the 4^{th} order polynomial system is a valuable attribute. Overall the 4^{th} order polynomial system did a good job in minimizing the losses due to the inevitable whipsaws that will occur in any trading system and maximizing the profits from the major intraday trend moves of the S&P contract.

In order to use this system in real time trading, at least ten more test and out-of-sample windows would have to be examined to make sure that the above results above were not due to pure chance.

References:

Meyers, Dennis , "The British Pound, Cubed", *Stocks & Commodities*, Volume 16: November.1998

Info on Dennis Meyers

Dennis Meyers has a doctorate in applied mathematics in engineering. He is a member of the Chicago Board Options Exchange(CBOE), a private trader, and president of Meyers Analytics. His firm specializes in consulting for financial institutions and developing publicly available analytical software for traders. He can be reached (312) 280-1687, via his Web site at http://www.MeyersAnalytics.com or via E-mail at support@MeyersAnalytics.com.

Figure 2 Test Window Performance Summary for S&P Sep/99 4th Order Polynomial System 07/07/99 - 08/05/99

4th Order Polynomial System S&P Sep/99 5 min 07/07/99 - 08/05/99.

Performance Summary: All Trades

Total net profit	$ 20900.000	Open position P/L	$ 0.000
Gross profit	$ 35625.000	Gross loss	$-14725.000
Total # of trades	30	Percent profitable	63%
Number winning trades	19	Number losing trades	11
Largest winning trade	$ 5675.000	Largest losing trade	$ -2850.000
Average winning trade	$ 1875.000	Average losing trade	$ -1338.636
Ratio avg win/avg loss	1.401	Avg trade(win & loss)	$ 696.667
Max consec. winners	7	Max consec. losers	4
Avg # bars in winners	53	Avg # bars in losers	50
Max intraday drawdown	$ -6900.000		
Profit factor	2.419	Max # contracts held	1

Performance Summary: Long Trades

Total net profit	$ 300.000	Open position P/L	$ 0.000
Gross profit	$ 8700.000	Gross loss	$ -8400.000
Total # of trades	10	Percent profitable	60%
Number winning trades	6	Number losing trades	4
Largest winning trade	$ 3550.000	Largest losing trade	$ -2850.000
Average winning trade	$ 1450.000	Average losing trade	$ -2100.000
Ratio avg win/avg loss	0.690	Avg trade(win & loss)	$ 30.000
Max consec. winners	3	Max consec. losers	3
Avg # bars in winners	55	Avg # bars in losers	35
Max intraday drawdown	$ -8425.000		
Profit factor	1.036	Max # contracts held	1

Performance Summary: Short Trades

Total net profit	$ 20600.000	Open position P/L	$ 0.000
Gross profit	$ 26925.000	Gross loss	$ -6325.000
Total # of trades	20	Percent profitable	65%
Number winning trades	13	Number losing trades	7
Largest winning trade	$ 5675.000	Largest losing trade	$ -2250.000
Average winning trade	$ 2071.154	Average losing trade	$ -903.571
Ratio avg win/avg loss	2.292	Avg trade(win & loss)	$ 1030.000
Max consec. winners	6	Max consec. losers	5
Avg # bars in winners	52	Avg # bars in losers	58
Max intraday drawdown	$ -4275.000		
Profit factor	4.257	Max # contracts held	1

Figure 3 Out-Of-Sample Performance Summary for S&P Sep/99 4th Order Polynomial System 08/06/99 - 08/13/99

4th Order Polynomial System S&P Sep/99 5 min 08/06/99 - 08/13/99.

Performance Summary: All Trades

Total net profit	$ 6475.000	Open position P/L	$ 0.000
Gross profit	$ 10950.000	Gross loss	$ -4475.000
Total # of trades	10	Percent profitable	70%
Number winning trades	7	Number losing trades	3
Largest winning trade	$ 3625.000	Largest losing trade	$ -2625.000
Average winning trade	$ 1564.286	Average losing trade	$ -1491.667
Ratio avg win/avg loss	1.049	Avg trade(win & loss)	$ 647.500
Max consec. winners	4	Max consec. losers	2
Avg # bars in winners	46	Avg # bars in losers	44
Max intraday drawdown	$ -4725.000		
Profit factor	2.447	Max # contracts held	1

Performance Summary: Long Trades

Total net profit	$ 2950.000	Open position P/L	$ 0.000
Gross profit	$ 4800.000	Gross loss	$ -1850.000
Total # of trades	5	Percent profitable	60%
Number winning trades	3	Number losing trades	2
Largest winning trade	$ 3000.000	Largest losing trade	$ -1675.000
Average winning trade	$ 1600.000	Average losing trade	$ -925.000
Ratio avg win/avg loss	1.730	Avg trade(win & loss)	$ 590.000
Max consec. winners	1	Max consec. losers	1
Avg # bars in winners	38	Avg # bars in losers	47
Max intraday drawdown	$ -1875.000		
Profit factor	2.595	Max # contracts held	1

Performance Summary: Short Trades

Total net profit	$ 3525.000	Open position P/L	$ 0.000
Gross profit	$ 6150.000	Gross loss	$ -2625.000
Total # of trades	5	Percent profitable	80%
Number winning trades	4	Number losing trades	1
Largest winning trade	$ 3625.000	Largest losing trade	$ -2625.000
Average winning trade	$ 1537.500	Average losing trade	$ -2625.000
Ratio avg win/avg loss	0.586	Avg trade(win & loss)	$ 705.000
Max consec. winners	3	Max consec. losers	1
Avg # bars in winners	53	Avg # bars in losers	37
Max intraday drawdown	$ -3050.000		
Profit factor	2.343	Max # contracts held	1

FIGURE 4 TradeStation 2000i TRADE BY TRADE SUMMARY
S&P Sep/99 5min 4th Order Polynomial System 08/06/99 - 08/13/99

Trade # Type	Date	Time	Price	Contracts Profit	% Profit Cum Profit	Run-up Drawdown
1	8/6/99	10:55	1312.30	1	0.40%	3525.00
Sell	8/6/99	16:15	1307.00	1325.00	1325.00	-1050.00
2	8/9/99	09:50	1306.00	1	0.23%	1350.00
Sell	8/9/99	16:15	1303.00	750.00	2075.00	-1700.00
3	8/10/99	09:40	1301.50	1	1.11%	7375.00
Sell	8/10/99	15:05	1287.00	3625.00	5700.00	-750.00
4	8/10/99	15:05	1287.00	1	0.27%	1375.00
Buy	8/10/99	16:15	1290.50	875.00	6575.00	-1000.00
5	8/11/99	09:40	1296.50	1	-0.52%	875.00
Buy	8/11/99	11:20	1289.80	-1675.00	4900.00	-1875.00
6	8/11/99	11:20	1289.80	1	-0.81%	300.00
Sell	8/11/99	14:25	1300.30	-2625.00	2275.00	-3050.00
7	8/11/99	14:25	1300.30	1	0.28%	1800.00
Buy	8/11/99	16:15	1304.00	925.00	3200.00	-75.00
8	8/12/99	09:40	1307.20	1	-0.05%	3075.00
Buy	8/12/99	15:50	1306.50	-175.00	3025.00	-550.00
9	8/12/99	15:50	1306.50	1	0.14%	1125.00
Sell	8/12/99	16:15	1304.70	450.00	3475.00	-125.00
10	8/13/99	09:50	1321.30	1	0.91%	3050.00
Buy	8/13/99	16:15	1333.30	3000.00	6475.00	-375.00

FIGURE 5a S&P Sep/99 5min Bar Chart 4th Order Polynomial System 08/06/99 - 08/13/99

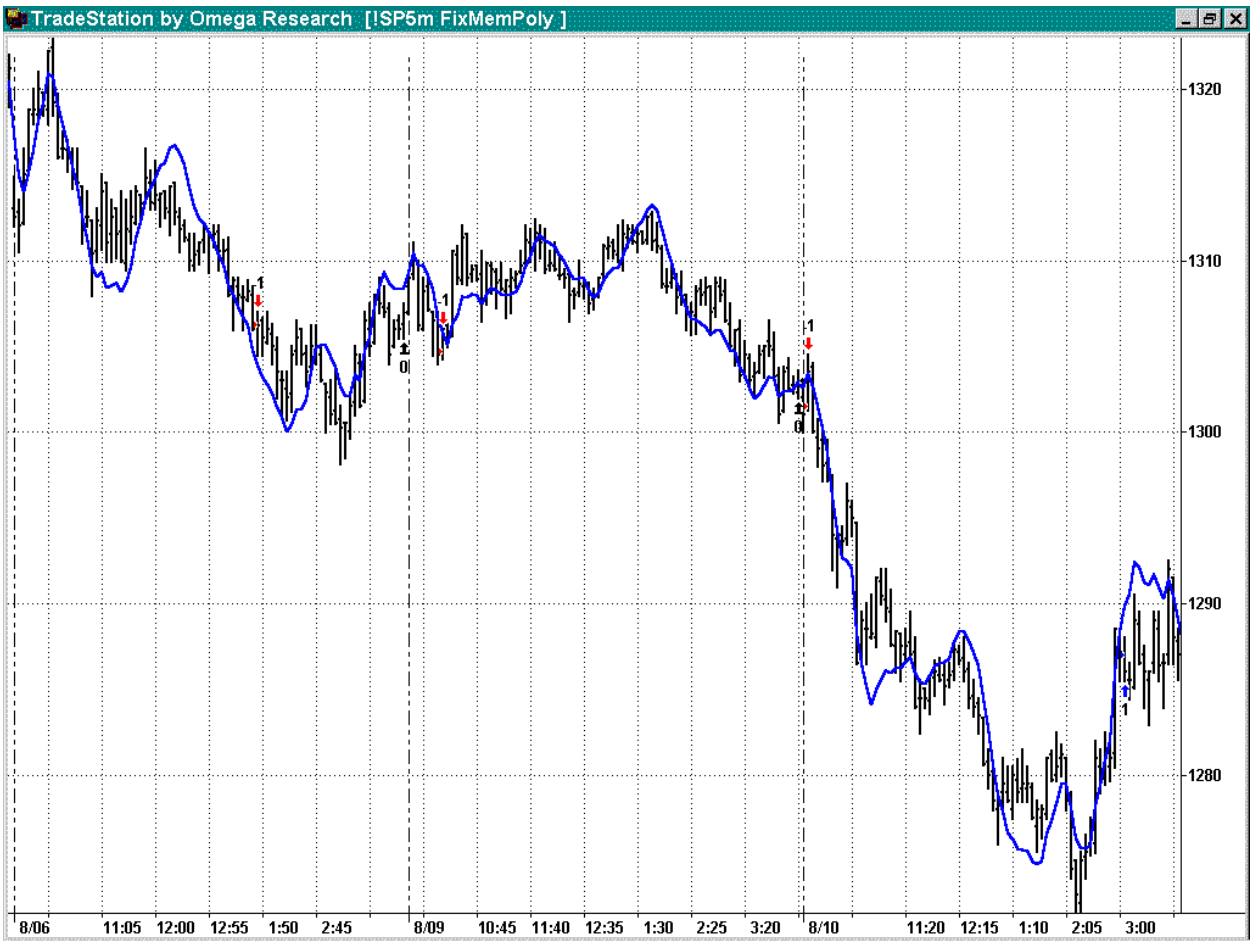

FIGURE 5b S&P Sep/99 5min Bar Chart 4th Order Polynomial System 08/06/99 - 08/13/99

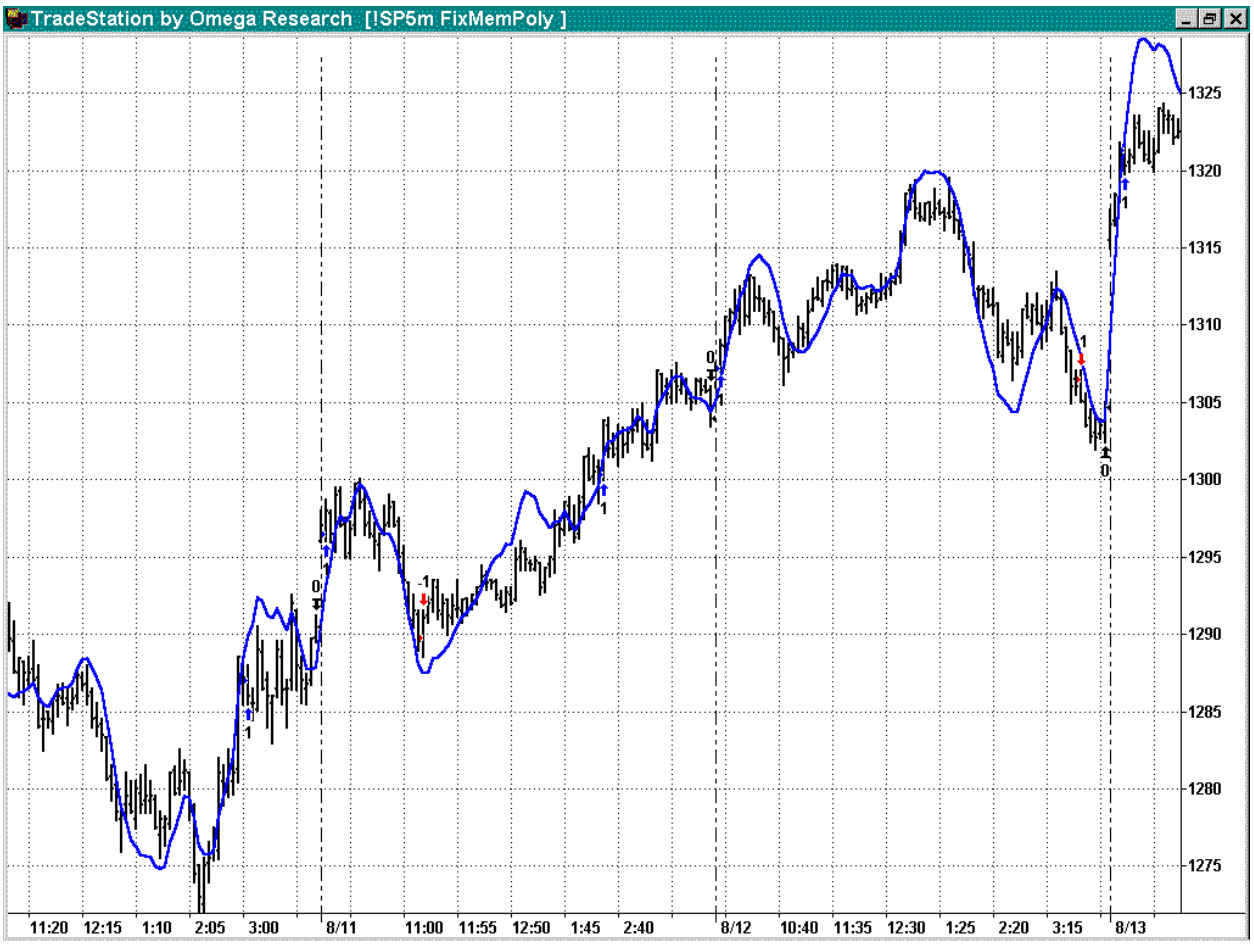

FIGURE 5c S&P Sep/99 5min Bar Chart 4th Order Polynomial System 08/06/99 - 08/13/99