

The n^{th} Order Adaptive Polynomial Acceleration System Applied to Japanese Yen Futures 1min Bars

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In previous working papers we showed how the application of a price curve generated by second, third and fourth degree polynomial velocity could be used to develop a strategy to buy and sell intraday futures. The reasoning behind this type of system was to only trade when the price trend velocity was above a certain threshold. Many times prices are meandering around without a notable trend. At these times we do not wish to trade because of the whipsaws losses that occur from this type of price action. When a price trend finally starts, the velocity of that price trend moves above some minimum value. Thus the velocity system would only issue a trade when certain velocity thresholds were crossed. Acceleration is the change in velocity per unit time. In order for velocity to reach a certain value the price changes must start accelerating. Thus acceleration should be an even earlier indicator than velocity that prices are starting to trend. Here we will use the walk forward optimization to determine the “best” polynomial order and use the Fixed Memory Polynomial Acceleration System to trade the Japanese Yen futures contract on an intraday basis using one minute bar price data. To test this system we will use one minute bar prices of the Japanese Yen futures contract traded on the CME and known by the symbol JY from February 24, 2005 to March 24, 2006.

The n^{th} Order Fixed Memory System Defined

The least squares forecast n^{th} order fixed memory polynomial velocity is constructed by solving for the coefficients $\beta_0, \beta_1, \beta_2, \beta_3 \dots \beta_n$ for the discrete orthogonal Legendre polynomials each day using the last **N bars** of closing prices and the equation for β_j shown in the “Math” appendix at the end of this working paper. Then **Acceleration(T+1)** is constructed from the equation shown in the “Math” appendix and plotted under the price chart.

The acceleration of a 2nd, 3rd and 4th order polynomial curve should change faster than its corresponding polynomial curve velocity. Whether higher order polynomial accelerations are an advantage or not, we will let the computer decide when we use a walk forward optimization technique described below.

At each bar we calculate the n^{th} order (2nd through 4th) fixed memory polynomial acceleration from the formulas in the “Math” appendix. As we will show below, optimization will determine the order for n^{th} order polynomial acceleration that will be used. When the acceleration is greater than the threshold amount **aup** we will go long. When the acceleration is less than the threshold amount **-adn** we will go short.

Buy Rule:

IF Acceleration is greater than the threshold amount **aup** then buy at the market.

Sell Rule:

IF Acceleration is less than the threshold amount **-adn** then sell at the market.

Intraday Bars Exit Rule:

Close the position 5 minutes before the JY close (no trades will be carried overnight).

Intraday Bars First Trade of Day Entry Rule:

- Ignore all trade signals before **XOpen** minutes after the open. For the Buy and Sell rules above we have included a first trade of the day entry rule. We’ve included this rule because often there are gaps in the open creating immediate system buys and sells. Many times these gaps are closed creating a losing whipsaw trade. In order to avoid the opening gap whipsaw trade problem we’ve delayed the first trade of the day for **20** minutes after the CME open.

Discussion of Japanese Yen Prices

The Japanese Yen (JY) is traded on Globex and on the trading floor at the CME. On Globex the JY is traded on a 23hour basis. The CME hours for floor trading (RTH) are 7:20 to 14:00 CST. Over 50% of the volume in the JY is

done on Globex is during the CME RTH hours. We have restricted our study to only trading the JY during the CME RTH 7:20 to 1400 hours.

Testing The Polynomial Acceleration System Using Walk Forward Optimization

There will be four strategy parameters to determine:

1. **degree**, degree=1 for straight line acceleration, degree=2 for 2nd order acceleration, etc.
2. **N**, is the number of lookback bars of prices to calculate the **acceleration**.
3. **aup**, the threshold amount that acceleration has to be greater than to issue a buy signal
4. **adn**, the threshold amount that acceleration has to be less than to issue a sell signal
5. **Xopn**, the number of minutes after the opening until the first trade of the day can execute.

To test this system we will use one minute bar prices of the Japanese Yen (JY) futures contract traded on the CME and known by the symbol JY from November 4, 2004 to November 19, 2005 and traded during the CME RTH session from 7:20 to 14:00 CST.

We will test this strategy with the above JY 1 min bars on a walk forward basis, as will be described below. To create our walk forward files we will also use the **add-in** software product called the Power Walk Forward Optimizer (PWFO). In TradeStation (TS), we will run the PWFO strategy **add-in** along with the nth Order Polynomial Acceleration Strategy on the JY 1min data from 2/24/2005 to 3/24/2006. The PWFO will breakup and create fifty-two 30 calendar day test sections along with their corresponding 7 calendar day out-of-sample sections from the one year of JY TS optimization (see Walk forward Testing below). The test and out-of-sample section dates are shown in **Table 1** on page 7 below. We will then use another software product called the Walk Forward Performance Explorer (WFPE) on each of the fifty-two test and out-of-sample(oos) sections generated by the PWFO to find the best test section performance **filter** that determines the system input parameters (**degree, N, aup, adn**) that will be used on the out-of-sample data. Detailed information about the PWFO and the WFPE can be found at www.meyersanalytics.com

For the test data we will run the TradeStation optimization engine on the one year of JY 1 min bars with the following ranges for the nth order polynomial acceleration strategy inputs variables.

1. degree from 2 to 4
2. N from 20 to 70 in steps of 10.
3. aup from 1 to 4 steps of 0.25
4. adn from 1 to 4 in steps of 0.25

This will produce 3042 different cases or combinations of the input parameters for each of the 52 PWFO output files.

Walk Forward Out-of-Sample Testing

Walk forward analysis attempts to minimize the curve fitting price noise by using the law of averages from the Central Limit Theorem on the out-of-sample performance. In walk forward analysis the data is broken up into many test and out-of-sample sections. Usually for any system one has some performance parameter selection procedure which we will call a **filter** used to select the input parameters from the optimization run. For instance, a **filter** might be all cases that have a profit factor (PF) greater than 1 and less than 3. For the number of cases left we might select the cases that had the best percent profit. This procedure would leave you with one case in the test section output and its associated strategy input parameters. Now suppose we ran our optimization on each of our many test sections and applied our filter to each test section output. We would then use the strategy input parameters found by the filter in each test section on the out-of-sample section immediately following that test section. The input parameters found in each test section and applied to each out-of-sample section would produce independent net profits and losses for each of the out-of-sample sections. Using this method we now have "x" number of independent out-of-sample section profit and losses from our filter. If we take the average of these out-of-sample section net profits and losses than we will have an estimate of how our system will perform on average. Due to the Central Limit Theorem, as our sample size increases, the spurious noises results in the out-of-sample section performance should average out to zero in the limit leaving us with what to expect from our system and filter. Mathematical note: This assumption assumes that the out-of-sample returns are from probability distributions that have a finite variance.

Why use the walk forward technique? Why not just perform an optimization on the whole price series and choose the input parameters that give the best total net profits or profit factor? Surely the price noise cancels itself out with such a large number of test prices and trades. Unfortunately, nothing could be farther from the truth! Optimization is a misnomer and should really be called combinatorial search. As stated above, whenever we run a combinatorial search over many different combinations of input parameters on noisy data on a fixed number of prices, **no matter how many**, the best performance parameters found are guaranteed to be due to “curve fitting” the noise and signal. What do we mean by “curve fitting”? Price series that we trade consists of random spurious price movements, which we call noise, and repeatable price patterns (*if they exist*). When we run, say, 8000 different input parameter combinations, the best performance parameters will be from those system input variables that are able to produce profits from the price pattern **and** the random spurious movements. While the price patterns will repeat, the same spurious price movements will not. If the spurious movements that were captured by a certain set of input parameters were a large part of the total net profits, then choosing these input parameters will produce losses when traded on future data. These losses occur because the spurious movements will not be repeated in the same way. This is why system optimization or combinatorial searches with no out-of-sample testing cause losses when traded in real time from something that looked great in the test section. Unfortunately it is human nature to extrapolate past performance to project future trading results and so results from curve fitting give the illusion, a modern “siren call” so to speak, of future trading profits.

In order to gain confidence that our input parameter selection method using the optimization output of the test data will produce profits, we must test the input parameters we found on out-of-sample data. In addition, we must perform the test/out-of-sample analysis many times. Why not just do the out-of-sample analysis once? Well just as in Poker or any card game, where there is considerable vagaries in hand to hand luck, walk forward out-of-sample analysis give considerable vagaries in week to week out-of-sample profit “luck”. That is, by pure chance we may have chosen some input parameter set that did well in the test section data **and** the out-of-sample section data. In order to minimize this type of “luck”, statistically, we must repeat the walk forward out-of-sample (oos) analysis over many test/oos sections and take an average of our weekly results over all out-of-sample sections. This average gives us an expected weekly return and a standard deviation of weekly returns which allows us to statistically estimate the expected equity and it’s range for N weeks in the future.

Finding The Strategy Input Parameters in The Walk Forward Test Sections

What is a performance filter? The only thing we know for sure is that the curve fitting of the signal and noise will always produce the best performance results. Thus if we eliminate the optimization cases with the very best performance results we are sure to eliminate many of the data mining system input parameters that fitted the past spurious random price movements. We know from experience that very few good intraday systems can sustain profit factors(PF) above 3 or 4 over time. If we eliminate, or *filter*, from our optimization, all cases that have profit factors greater than 3 or 4 then we will probably eliminate most of the cases that are due to curve fitting the noise. In addition, since in real time it is tough to sustain more than five losses in a row(LR) and still keep trading, we will take the extra precaution and eliminate all those cases that have more than five losses in a row.

To find a performance filter on the test section, we will use a product named the Walk Forward Performance Explorer (WFPE) on each of the 52 PWFO test/oos files that were generated by using TS’s combinatorial optimization module on the years worth of JY 1min bar data with the PWFO strategy add-in. The WFPE examines the out-of-sample performance of various combinations of PF from 2 to 4 and LR from 3 to 5 filters eliminating test sample cases for each PF-LR combination examined. The WFPE then calculates and reports the average out-of-sample performance of all 52 oos files for each filter examined.

The WFPE revealed that the following filter will produce the most consistent and reliable out-of-sample results..

Filter: PF<=3.5 and LR<=5 and #Trds>=10 and minimum nT

Where:

- PF = Profit Factor in Test optimization section

- **LR** = Maximum consecutive losses in a row in test optimization section. Since in real time it is tough to sustain more than five losses in a row and still keep trading, we will eliminate all those cases that have more than five losses in a row
- **#Trds** = The number of trades in the test run. We want our system to trade almost every day. There are 20 to 23 trading days a month, so we only want to look at input parameters that are able to generate at least 12 trades in every one month test sample.
- **nT** = Minimum number of trades in test section but ≥ 10

The first part of the filter chooses those rows or cases out of the 3042 rows in each PWFO file test (in-sample) section that satisfy the criteria **PF** ≤ 3.5 and **LR** ≤ 5 and **#Trds** ≥ 10 . This part of the filter will leave anywhere from 0 to 200 rows out of the 3042 rows that satisfy this filter criteria. Next we find the one row that has the smallest **nT** ≥ 10 . This **Filter** or selection procedure will leave only one choice for the system input values of *degree*, *N*, *aup*, *adn*. We then use these input values found in the test section by the **Filter** on the **next week** of one minute bar JY **out-of-sample** price bars **following** the test section.

Results

Table 1 on page 8 below presents a table of the fifty-two test and out-of-sample windows, the selected optimum parameters and the weekly out-of-sample results using the filter described above.

Figure 1 presents the out-of-sample 1 minute bar chart of JY for 3/24/2006. with the Acceleration Indicator and all the buy and sell signals for that date.

Discussion of System Performance

At the bottom of **Figure 1** are some statistics that are of interest. In the “Averages” row #53, the first statistic OSNP Average=\$176, is the average weekly net profit for the fifty-two out-of-sample weeks and ONT Average= 2.49 is the average number of trades per week. The OSNP Std Dev =\$342 is the standard deviation of the weekly return. Given a process that generates the above average and standard deviation, it would be interesting to know, statistically, the probabilistic outcome from trading this system for 13 weeks (one quarter). There is a statistical technique called bootstrap Ref [1]. Using the bootstrap technique we would randomly choose 13 of the fifty weekly profits in Table 1, with replacement and take the sum of those 13 randomly chosen out-of-sample profits. “With replacement” means we don’t eliminate the randomly chosen weekly profit from being chosen again. Let us suppose that we repeat this random choosing of 13 weeks and summing the results 200 times. We would then have 200 different 13 week net profit summations. If we take the average and standard deviation of those 200 different 13 week net profit summations we would have an estimate of what to expect from this system by trading it for 13 weeks.

For this system the 200 bootstrap average of 13 week net profits is \$2274 and the standard deviation is \$1269. This means that at two standard deviations we can expect our 13 week return from this system to be between -\$264 and \$4812 95% of the time. This is quite a range and indicates that 13 weeks of trading is not enough time to judge the results of trading this system. Assuming that the bootstrap averages are from a normal distribution an interesting number is called the 99% breakeven time. This is how many weeks do we need to trade this system so that we have a 99% probability that the equity after those number of weeks will be greater than zero. The answer is 16 weeks. That is, there is less than a 1% chance that our equity will be negative after 16 weeks of trading. Please note that slippage and commissions were not taken into account so the numbers obtained above are higher than could be attained from actual trading.

To see the effect of walk forward analysis take a look at **Table 1**. Notice how the input parameters *pw*, *N*, *aup* and *adn* take sudden jumps from high to low and back. This is the walk forward process quickly adapting to changing volatility conditions in the test sample. In addition, notice how often *degree* changes from a parabolic line acceleration with *degree*=2 to a 3rd and 4th order acceleration with *degree*= 3 and 4. The 3rd and 4th order accelerations, due to the higher order components component, change much faster than the parabolic line acceleration. When the data gets very noisy with a lot of spurious price movements it’s better to have the acceleration change slower filtering out the noisy data. During other times when the noise level is not as much it is better to have the acceleration break its *aup* and *adn* barriers faster to get onboard a trend faster. This is what the

filter is doing. When there is a lot of noise in the test section it switches to the 2nd order curve acceleration. When the noise level is lower in the test section it switches to the faster changing 3rd or 4th order curve acceleration.

Using this filter, the strategy was able to generate \$8602 (equity) trading one JY contract. No positions were carried over night. The largest losing week was -\$550 and the largest drawdown was -\$1127. The longest time between new equity highs was 9 weeks.

In observing Table 1 we can see that this strategy and filter did not trade much each week. The strategy seemed to wait for really strong trends and then initiate a buy or sell. There were many weeks with just one trade. There were three weeks with no trades at all. In observing the chart for 3/24/06 we can see this trading only a big trend action. For the week of 3/20/06 to 3/24/06 there was only one trade for the week on 3/24/06.

References

1. Efron, B., Tibshirani, R.J., (1993), "An Introduction to the Bootstrap", New York, Chapman & Hall/CRC.
2. Morrison, Norman "Introduction to Sequential Smoothing and Prediction", McGraw-Hill Book Company, New York, 1969.

Table 1 Walk Forward Out-Of-Sample Performance Summary for JY Polynomial Acceleration System

JY-1 min bars 2/24/2005 - 3/24/2006. The input values *degree(pw)*, *N*, *aup*, *adn* are the values found from applying the filter to the test Sample optimization runs.

Filter= PF<3.5 and LR<=5 and #Trds>=10 and minimum nT

Week	Test Dates	Out-Of-Sample Dates	osnp	Equity	ollt	odd	ont	avosnp	pw	N	aup	adn	xop	xt
1	02/23/05 To 03/25/05	03/28/05 To 04/01/05	200.0	200.0	(525.0)	(625.0)	4	50.0	3	20	2.5	2	20	1350
2	03/02/05 To 04/01/05	04/04/05 To 04/08/05	438.0	638.0	(125.0)	(125.0)	4	109.5	4	40	3.25	2.5	20	1350
3	03/09/05 To 04/08/05	04/11/05 To 04/15/05	113.0	751.0	0.0	0.0	2	56.5	2	40	3.25	3.25	20	1350
4	03/16/05 To 04/15/05	04/18/05 To 04/22/05	163.0	914.0	(225.0)	(225.0)	2	81.5	2	40	2.75	3.25	20	1350
5	03/23/05 To 04/22/05	04/25/05 To 04/29/05	63.0	977.0	(150.0)	(150.0)	2	31.5	2	40	2.75	3.25	20	1350
6	03/30/05 To 04/29/05	05/02/05 To 05/06/05	350.0	1327.0	(13.0)	(13.0)	2	175.0	2	40	3.25	2.75	20	1350
7	04/06/05 To 05/06/05	05/09/05 To 05/13/05	(175.0)	1152.0	(288.0)	(288.0)	3	(58.3)	2	40	2.75	3.25	20	1350
8	04/13/05 To 05/13/05	05/16/05 To 05/20/05	413.0	1565.0	0.0	0.0	1	413.0	2	20	2	3	20	1350
9	04/20/05 To 05/20/05	05/23/05 To 05/27/05	38.0	1603.0	0.0	0.0	1	38.0	4	20	3.25	2.25	20	1350
10	04/27/05 To 05/27/05	05/30/05 To 06/03/05	(313.0)	1290.0	0.0	0.0	1	(313.0)	2	50	2.75	3.5	20	1350
11	05/04/05 To 06/03/05	06/06/05 To 06/10/05	(138.0)	1152.0	0.0	0.0	1	(138.0)	4	20	2.5	2.25	20	1350
12	05/11/05 To 06/10/05	06/13/05 To 06/17/05	(263.0)	889.0	0.0	0.0	1	(263.0)	3	50	3.25	2.5	20	1350
13	05/18/05 To 06/17/05	06/20/05 To 06/24/05	(63.0)	826.0	(213.0)	(313.0)	5	(12.6)	4	60	3.25	2.5	20	1350
14	05/25/05 To 06/24/05	06/27/05 To 07/01/05	(175.0)	651.0	(75.0)	(175.0)	3	(58.3)	4	70	3.25	2.5	20	1350
15	06/01/05 To 07/01/05	07/04/05 To 07/08/05	(175.0)	476.0	0.0	0.0	1	(175.0)	4	30	3.25	3.25	20	1350
16	06/08/05 To 07/08/05	07/11/05 To 07/15/05	750.0	1226.0	0.0	0.0	2	375.0	4	30	3.25	2.5	20	1350
17	06/15/05 To 07/15/05	07/18/05 To 07/22/05	(413.0)	813.0	(525.0)	(700.0)	6	(68.8)	4	30	3.5	2.5	20	1350
18	06/22/05 To 07/22/05	07/25/05 To 07/29/05	725.0	1538.0	0.0	0.0	4	181.2	2	60	2.25	3.25	20	1350
19	06/29/05 To 07/29/05	08/01/05 To 08/05/05	850.0	2388.0	(25.0)	(25.0)	5	170.0	2	60	2.5	3.25	20	1350
20	07/06/05 To 08/05/05	08/08/05 To 08/12/05	275.0	2663.0	(13.0)	(13.0)	2	137.5	3	20	2.25	3.25	20	1350
21	07/13/05 To 08/12/05	08/15/05 To 08/19/05	350.0	3013.0	(88.0)	(88.0)	2	175.0	3	20	2	3.25	20	1350
22	07/20/05 To 08/19/05	08/22/05 To 08/26/05	250.0	3263.0	0.0	0.0	1	250.0	4	20	2.25	3.5	20	1350
23	07/27/05 To 08/26/05	08/29/05 To 09/02/05	100.0	3363.0	(450.0)	(450.0)	3	33.3	2	40	2.75	2.25	20	1350
24	08/03/05 To 09/02/05	09/05/05 To 09/09/05	575.0	3938.0	0.0	0.0	1	575.0	3	60	3	3.25	20	1350
25	08/10/05 To 09/09/05	09/12/05 To 09/16/05	463.0	4401.0	(150.0)	(150.0)	4	115.8	2	70	3	3.25	20	1350
26	08/17/05 To 09/16/05	09/19/05 To 09/23/05	275.0	4676.0	0.0	0.0	1	275.0	2	40	3.25	3	20	1350
27	08/24/05 To 09/23/05	09/26/05 To 09/30/05	(38.0)	4638.0	(100.0)	(100.0)	2	(19.0)	2	40	2.5	3.25	20	1350
28	08/31/05 To 09/30/05	10/03/05 To 10/07/05		4638.0					2	30	3	3	20	1350
29	09/07/05 To 10/07/05	10/10/05 To 10/14/05	(550.0)	4088.0	0.0	0.0	1	(550.0)	2	20	3	1.75	20	1350
30	09/14/05 To 10/14/05	10/17/05 To 10/21/05	(225.0)	3863.0	(150.0)	(238.0)	4	(56.2)	3	70	3.25	2	20	1350
31	09/21/05 To 10/21/05	10/24/05 To 10/28/05	300.0	4163.0	(125.0)	(150.0)	4	75.0	2	70	2.5	2.25	20	1350
32	09/28/05 To 10/28/05	10/31/05 To 11/04/05	(38.0)	4125.0	0.0	0.0	1	(38.0)	4	20	2.25	2.75	20	1350
33	10/05/05 To 11/04/05	11/07/05 To 11/11/05	(275.0)	3850.0	(313.0)	(313.0)	2	(137.5)	3	20	2.5	1.75	20	1350
34	10/12/05 To 11/11/05	11/14/05 To 11/18/05	238.0	4088.0	0.0	0.0	2	119.0	2	20	2.75	1.5	20	1350
35	10/19/05 To 11/18/05	11/21/05 To 11/25/05	625.0	4713.0	0.0	0.0	1	625.0	3	50	3.5	2.75	20	1350
36	10/26/05 To 11/25/05	11/28/05 To 12/02/05	125.0	4838.0	(50.0)	(50.0)	3	41.7	3	70	2.5	2.75	20	1350
37	11/02/05 To 12/02/05	12/05/05 To 12/09/05		4838.0					3	20	2	2.75	20	1350
38	11/09/05 To 12/09/05	12/12/05 To 12/16/05	625.0	5463.0	(350.0)	(375.0)	7	89.3	3	40	2.75	3.25	20	1350
39	11/16/05 To 12/16/05	12/19/05 To 12/23/05	(338.0)	5125.0	(500.0)	(500.0)	2	(169.0)	3	60	3	3	20	1350
40	11/23/05 To 12/23/05	12/26/05 To 12/30/05	325.0	5450.0	(63.0)	(63.0)	2	162.5	3	30	3.25	3.25	20	1350
41	11/30/05 To 12/30/05	01/02/06 To 01/06/06	300.0	5750.0	(138.0)	(163.0)	4	75.0	4	20	3	2.5	20	1350
42	12/07/05 To 01/06/06	01/09/06 To 01/13/06	363.0	6113.0	0.0	0.0	1	363.0	2	60	3.5	2.5	20	1350
43	12/14/05 To 01/13/06	01/16/06 To 01/20/06	413.0	6526.0	(213.0)	(213.0)	4	103.2	4	50	3.25	3.5	20	1350
44	12/21/05 To 01/20/06	01/23/06 To 01/27/06	888.0	7414.0	0.0	0.0	3	296.0	4	30	3	2.5	20	1350
45	12/28/05 To 01/27/06	01/30/06 To 02/03/06	(100.0)	7314.0	0.0	0.0	1	(100.0)	4	40	3.25	3.5	20	1350
46	01/04/06 To 02/03/06	02/06/06 To 02/10/06	288.0	7602.0	0.0	0.0	1	288.0	3	20	3.5	2	20	1350

47	01/11/06 To 02/10/06	02/13/06 To 02/17/06	325.0	7927.0	(13.0)	(13.0)	2	162.5	2	70	3	3.25	20	1350
48	01/18/06 To 02/17/06	02/20/06 To 02/24/06	263.0	8190.0	0.0	0.0	3	87.7	2	60	2.75	3.5	20	1350
49	01/25/06 To 02/24/06	02/27/06 To 03/03/06	25.0	8215.0	(63.0)	(63.0)	3	8.3	4	30	3.5	3.5	20	1350
50	02/01/06 To 03/03/06	03/06/06 To 03/10/06		8215.0					3	70	3.25	3.25	20	1350
51	02/08/06 To 03/10/06	03/13/06 To 03/17/06	(188.0)	8027.0	(575.0)	(575.0)	4	(47.0)	2	50	2.75	2.75	20	1350
52	02/15/06 To 03/17/06	03/20/06 To 03/24/06	575.0	8602.0	0.0	0.0	1	575.0	2	40	2	3.5	20	1350
	Averages		176.0					2.49	70.5					
	Standard Deviation		342.0											
	200Boot 13wk Sum Avg		2274.0											
	200 Boot 13wk Sum Std		1269.0											

PF = Profit Factor in Test optimization section

LR= Maximum consecutive losses in a row in test optimization section

#Trds = Minimum number of trades in the test section.

nT= Ratio (Total winning bars)/(Total losing bars) in Test Section

osnp = Weekly Out-of-sample net profit in \$

Equity = running sum of the weekly out-of-sample net profits in \$

ollt = The largest losing trade in the out-of-sample section in \$.

odd = The close drawdown in the out-of-sample section in \$.

ont = The number of trades in the out-of-sample week.

avosnp = The average profit per trade for that week \$

pw = polynomial degree. degree=2 for parabolic curve acceleration, degree=3 for 3rd order acceleration, etc.

N = is the number of bars lookback period to calculate the polynomial **acceleration**.

aup =, the threshold amount that acceleration has to be greater than to issue a buy signal

adn =, the threshold amount that acceleration has to be less than to issue a sell signal

xop = the number of minutes after the opening until the first trade of the day can execute.

Figure 2 Walk Forward Out-Of-Sample Performance Summary for JY Fixed Memory Polynomial Acceleration System 1 minute bar chart of JY from 3/24/06



Appendix: n^{th} Order Polynomial Next Bar's Forecast Math

What is the n^{th} Order Polynomial ?

The n^{th} Order Polynomial, also called the n^{th} Order Fixed Memory Polynomial, is simply the least square fit of a polynomial of the form $\mathbf{b}_0 + \mathbf{b}_1 * \mathbf{t} + \mathbf{b}_2 * \mathbf{t}^2 + \mathbf{b}_3 * \mathbf{t}^3 + \dots + \mathbf{b}_n * \mathbf{t}^n$ to a *fixed* number of past data points. Where \mathbf{t} is discrete time bars. Time could be daily bars or one minute bars. We use the term "Fixed Memory" to designate that only a fixed number of data points are used to calculate the polynomial coefficients. It is assumed that the time bars occur at fixed intervals of time so tic bars would not be appropriate for this analysis. Least squares is a mathematical technique where the squared vertical distance between the data and the curve that is being fit to the data is minimized. When the net squared distance (also called the sum of the squared errors) is minimized, a unique set of coefficients $\mathbf{b}_0, \mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_n$ for the polynomial is determined. This type of error minimization is mathematically solvable and is widely used in science and mathematics.

For a 4^{th} order polynomial equation, the least squares coefficients are obtained from the solution of the following matrix equation.

$$\begin{bmatrix} \mathbf{T} & \sum \mathbf{t} & \sum \mathbf{t}^2 & \sum \mathbf{t}^3 & \sum \mathbf{t}^4 \\ \sum \mathbf{t} & \sum \mathbf{t}^2 & \sum \mathbf{t}^3 & \sum \mathbf{t}^4 & \sum \mathbf{t}^5 \\ \sum \mathbf{t}^2 & \sum \mathbf{t}^3 & \sum \mathbf{t}^4 & \sum \mathbf{t}^5 & \sum \mathbf{t}^6 \\ \sum \mathbf{t}^3 & \sum \mathbf{t}^4 & \sum \mathbf{t}^5 & \sum \mathbf{t}^6 & \sum \mathbf{t}^7 \\ \sum \mathbf{t}^4 & \sum \mathbf{t}^5 & \sum \mathbf{t}^6 & \sum \mathbf{t}^7 & \sum \mathbf{t}^8 \end{bmatrix} \begin{bmatrix} \mathbf{a}_0 \\ \mathbf{b}_0 \\ \mathbf{c}_0 \\ \mathbf{d}_0 \\ \mathbf{e}_0 \end{bmatrix} = \begin{bmatrix} \sum \mathbf{p}(\mathbf{t}) \\ \sum (\mathbf{p}(\mathbf{t}) * \mathbf{t}) \\ \sum (\mathbf{p}(\mathbf{t}) * \mathbf{t}^2) \\ \sum (\mathbf{p}(\mathbf{t}) * \mathbf{t}^3) \\ \sum (\mathbf{p}(\mathbf{t}) * \mathbf{t}^4) \end{bmatrix}$$

where

$\mathbf{p}(\mathbf{T})$ is the current bar's price, $\mathbf{p}(\mathbf{T}-1)$ is the previous bar's price and $\mathbf{p}(\mathbf{1})$ is the price \mathbf{T} bars ago.

\mathbf{T} is the number of Bars in the Least Squares estimation

$\sum \mathbf{p}(\mathbf{t})$ is the summation of prices from $\mathbf{t}=1$ to \mathbf{T} bars

$\sum \mathbf{p}(\mathbf{t}) * \mathbf{t}$ is the summation of prices times \mathbf{t} from $\mathbf{t}=1$ to \mathbf{T} bars

$\sum \mathbf{t}$ is the summation of the integer \mathbf{t} from $\mathbf{t}=1$ to \mathbf{T} bars

$\sum \mathbf{t}^2$ is the summation of the integer \mathbf{t} squared from $\mathbf{t}=1$ to \mathbf{T} bars

etc.

Once the coefficients to the polynomial have been solved for we generate the forecast for the next bar's price which is given for the equation by:

$$\mathbf{P}_f = \mathbf{a}_0 + \mathbf{b}_0 * (\mathbf{T}+1) + \mathbf{c}_0 * (\mathbf{T}+1)^2 + \mathbf{d}_0 * (\mathbf{T}+1)^3 + \mathbf{e}_0 * (\mathbf{T}+1)^4$$

Where \mathbf{P}_f stands for price forecast.

With these coefficients, we can also generate the forecast for the next bar's *velocity* and *acceleration* by the equations:

$$\text{Velocity}(\mathbf{T}+1) = d\mathbf{P}_f / d\mathbf{t} = \mathbf{b}_0 + 2\mathbf{c}_0 * (\mathbf{T}+1) + 3\mathbf{d}_0 * (\mathbf{T}+1)^2 + 4\mathbf{e}_0 * (\mathbf{T}+1)^3$$

$$\text{Acceleration}(\mathbf{t}+1) = d^2\mathbf{P}_f / d^2\mathbf{t} = 2\mathbf{c}_0 + 6\mathbf{d}_0 * (\mathbf{T}+1) + 12\mathbf{e}_0 * (\mathbf{T}+1)^2$$

We use the next bar forecast because changes in the trend are more quickly reflected in the forecast price, velocity and acceleration than in the end point price, velocity and acceleration.

Programs that solve for the solution to matrix equations can be found in the book "Numerical Recipes" by W. Press, et. al. However these type of matrix equation solvers are very slow and for these type of problems are unstable.

They cause numerical errors and floating point overflows due to matrix inversion ill conditioning which produces false results.

Appendix: n^{th} Order Polynomial Next Bar's Forecast Math

Fortunately these type of problems can be solved by a fast, efficient and accurate algorithm using Discrete Orthogonal Legendre Polynomials. This method is explained in detail in Norman Morrison' book entitled "Introduction to Sequential Smoothing and Prediction", Chapter 7 page 223., referenced at the end of this section.

Without going into detail here (see Morrison reference), the polynomial filter can now be represented by:

$$P_e(t) = \sum_{j=0}^n \beta_j * \phi_j(t) \quad j=0 \text{ to } n$$

Where n is the polynomial order, T is the total number of Bars of data used in the Least Squares fit and

$$\beta_j = \sum_{k=0}^{T-1} p(t-T+k) * \phi_j(k)$$

$\phi_j(t)$ = the *normalized* discrete Legendre polynomial. t = an integer from 0 to T

The coefficients, $\beta_0, \beta_1, \beta_2, \beta_3, \dots, \beta_n$ for a n^{th} order polynomial can now be solved for by the equation above and we can generate the forecast for the next bar's close, velocity and acceleration which are given by the equations

$$P_F(T+1) = \beta_0 * \phi_0(T+1) + \beta_1 * \phi_1(T+1) + \beta_2 * \phi_2(T+1) + \beta_3 * \phi_3(T+1) + \dots + \beta_n * \phi_n(T+1)$$

$$\text{Velocity} = (dP_F/dt)_{(T+1)} = \beta_1 * (d\phi_1/dt)_{(T+1)} + \beta_2 * (d\phi_2/dt)_{(T+1)} + \beta_3 * (d\phi_3/dt)_{(T+1)} + \dots + \beta_n * (d\phi_n/dt)_{(T+1)}$$

$$\text{Acceleration} = (d^2P_F/d^2t)_{(T+1)} = \beta_2 * (d^2\phi_2/d^2t)_{(T+1)} + \beta_3 * (d^2\phi_3/d^2t)_{(T+1)} + \dots + \beta_n * (d^2\phi_n/d^2t)_{(T+1)}$$

The n^{th} Order Fixed Memory Forecast Next Bar's Acceleration System Defined

The least squares forecast is constructed by solving for the least squares coefficients $\beta_1, \beta_2, \dots, \beta_n$ at each bar using the last T bars of closing prices and the Discrete Orthogonal Legendre Polynomial equations for β_j above. Then **Acceleration** = $d^2P_F(T+1)/d^2t$ is constructed from the velocity equation above and plotted under the price chart. In general what we will be doing is following the plotted curve of **Acceleration** which is calculated at each bar from the previous T bars. When the velocity is greater than a threshold amount *aup* we will go long. When the velocity is less than a threshold amount *-adn* we will go short.

Buy Rule:

IF Acceleration is greater than the threshold amount *aup* then buy at the market.

Sell Rule:

IF Acceleration is less than the threshold amount *-adn* then sell at the market.

References

1. Morrison, Norman "Introduction to Sequential Smoothing and Prediction", McGraw-Hill Book Company, New York, 1969.