Trading The E-Mini Using The End Point Fast Fourier Transform

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In a previous article entitled "The Discrete Fourier Transform Illusion" we demonstrated the misuses of the mathematical technique called the Fourier Transform as applied to the S&P500 index. There we showed how fitting the Fourier Transform to the S&P500 index data series produced a perfect curve fit on past data giving the illusion that this technique would predict the major turning points of the S&P500 index. However, we demonstrated that when we examined the Fourier Transform on a day by day walk forward basis this seemingly wondrous predictive capability disappeared.

In the follow-up article entitled "The Endpoint Fast Fourier Transform System" we showed how to use the FFT in a technique called the End Point Fast Fourier Transform(EPFFT) to trade daily bars of the S&P 500 futures. Here it will shown how to use EPFFT technique on a walk forward basis to trade 1 minute bars of the E-mini futures.

The FFT Illusion Review

Figure 1 presents the noise filtered FFT, as discussed in my previous article, on the SP from January 16, 1998 to January 22,1999. The SP closing high was on July 20th, 1998. As can be seen from Figure 1 the FFT curve clearly leads the 7/20/1998 top and is pointing down on 7/20/98. This action gives the illusion that the FFT curve is a predictive indicator that leads the price series enabling one to escape the ensuing bear market drop. However, this FFT curve was generated on data from 1/16/98 to 1/22/99. What would our FFT curve look like if we generated our curve on July 20th, 1998?. Figure 2 presents the noise filtered FFT on the SP from 7/15/97 to 7/20/98. As can be seen from the noise filtered FFT curve generated on 7/20/1998 the curve is pointing straight up giving no indication what-so-ever of the coming large market drop. Why does this happen? When the FFT went to fit the data, it already knew where all the tops and bottoms were. The FFT mathematics minimize the error between the curve it generates and the real data points. This error minimization process forces the generated curve to smoothly fit the past data like a glove. As a matter of fact, it's almost impossible not to get an excellent fit.

The End Point FFT

In order to avoid the past data curve fit illusion, we will create an indicator that walks forward one bar at a time. This indicator will calculate the noise filtered FFT curve but only save the last point, or end point, of the curve on the day that it is calculated. We will then connect all the generated end points to produce a curve that matches what we would have seen if we performed the noise filtered FFT on the end point dates.

For example Figure 2 was calculated using data from 7/15/98 to 7/20/98. We would save the FFT curve point of 7/20/98 and slide our data window up one day from 7/16/98 to 7/21/98. We would calculate the noise filtered FFT of this new data window and save the FFT curve point of 7/21/98. We would keep sliding our data window forward one day at a time, calculate the noise filtered FFT and save the FFT curve end point. We would then connect all these saved endpoints to create a new curve which I have dubbed the End Point FFT or EPFFT for short. The EPFFT represents what we would observe in real time if we performed this procedure each day. Figure 3 represents such a noise filtered EPFFT curve. For reasons that we will explain later in this article, the EPFFT curve is on a different scale then that the SP prices. What's important

here is the shape of the EPFFT curve and where it's turning points occur. As we can observe from Figure 3, the EPFFT curve is quite different from the FFT curve in Figure 1. Instead of leading the SP closing price curve the EPFFT has anywhere from a zero day lag to a four day lag from the major tops and bottoms. The EPFFT had a zero day lag, peaking simultaneously with the SP futures at the futures top on 7/20/98.

EPFFT Construction Details

Unfortunately constructing the noise filtered FFT of a price data series is not quite as simple as just taking 512 closing prices, and directly plugging them into a FFT algorithm.

The mathematics of the Discrete Fourier Transform(DFT) assumes that the time-domain sample is periodic and that it has captured an integral number of periods. This means that the DFT assumes the end of the sampled series implicitly wraps around to the beginning to start all over again. Thus for 512 sampled data points the DFT assumes that these 512 sampled data points repeat every 512 samples. With real data series this is seldom the case and this creates what is called a wraparound effect in the frequency domain. The wraparound effect creates a lot of distortion on the ends of the price series when the noise filtered FFT is transformed back. Unfortunately the end point of the price series is the very point we wish to estimate. While we can do nothing about the wrap around violation, we can significantly lessen it's effects by what is called zero padding.

The DFT assumes the time domain sample is periodic and repeats. Suppose a price series starts at 400 and wiggles and wags for 512 data samples ending at the value of 600. The DFT assumes that the price series starts at zero, suddenly jumps to 400, goes to 600 and suddenly jumps down to zero again and then repeats. The DFT must create all kinds of different frequencies in the frequency domain to try and match this type of behavior. These false frequencies created to match the jumps and the high average price completely swamp the amplitudes of any real frequencies making them look like noise. Fortunately this effect can be almost eliminated by a simple technique called end point flattening.

The calculation of end point flattening coefficients is simple. If x(1) represents the first price in the sampled data series, x(n) represent the last point in the data series and y(i) equal to the new endpoint flattened series then:

$$\mathbf{a = x(1)\quad b=(x(n)-x(1))/(n-1)}$$

$$\mathbf{y(i) = x(i) - [a+ b*(i-1)] \quad for\ i=1\ to\ n \quad (1)}$$

We can see that when i=1 then y(1)=0 and when i=n then y(n) =0. What we've done is subtract the beginning value of the time series to make the first value equal to zero and then rotate the rest of the time series such that the end point is now zero. This technique reduces the endpoint distortion but introduces a low frequency artifact into the Fourier Frequency spectrum.

EPFFT Curve Construction

For this article a sliding time-bar window of 512 1 minute bars of the E-Mini Sep/02 futures from 6/12/02 to 7/12/02 will be used. The first time window will be the 512 price bars from 6/12/02 to 6/13/02.

Step 1 Take the Log_e of the 512 prices to minimize the exponential trend movements of the E-mini. End flatten these 512 log prices using equation (1) above.

Step 2 Take the FFT of these 512 points to create 512 frequency domain complex numbers $\mathbf{f_i}$.

Step 3 Use a Low Pass Threshold filter that zeros out the frequencies whose magnitudes are less than thres*Fmax. Thres is the decimal percentage of Fmax and Fmax is the maximum frequency amplitude of the spectrum.

Step 4 Do an inverse FFT on the noise filtered spectrum and save the last 2 points. Although it's not obvious yet, we will need the noise filtered point one sample before the endpoint.

Add back the end point flattening of **Step 1** and taking the exponential value to reverse the log_e

Step 5 Save the filtered end point and the point before. Call the endpoint ep(k) and the point before ep1(k) where k is the denotes the order of the sliding window. That is, the first sliding window k=1, the second, k=2, etc. Slide the 512 bar data window forward one bar, and repeat steps 1 through 4.

When the data window is moved forward one bar at a time a new data sample is added to the end and the data sample at the beginning is subtracted. This adding and subtracting causes the end point flattening coefficients and the power in the frequency spectrum to jump around creating distortion and jitter in the calculation of the noise filtered end point. This random jumping of the FFT endpoints as the data window slides forward in time adds a random jump to FFT end point curve. Fortunately this jumping can be minimized by creating a new end point curve from the two saved noise filtered end points, ep(k) and ep1(k), above in step 5. Since turning points are of interest rather than magnitude then in **step 5** a new variable will be created called **sumEP** where

sumEP(k) = sumEP(k-1) + ep(k)-ep1(k)

This new curve **sumEP(k)** is the sum of all the changes in the individual ep(k)'s from their noise filtered FFT value one sample before. This change series minimizes the magnitude jump problem creating a fairly smooth EPFFT curve.

The EPFFT System Defined

Even though sumEP is a fairly smooth curve as seen in Figure 3, it still has a number of short term wiggles preventing us from simply going long when the curve turns up and going short when the curve turns down. To create a system, we will use a simple curve following technique.

Buy Rule:

- **IF sumEP** has moved up by more than the point amount of *pup* from the lowest low recorded in **sumEP** while short then buy the ES Sep/02 futures at the market..

Sell Rule:

- **IF sumEP** has moved down by more than the point amount *pdn* from the highest high recorded in **sumEP** while long then sell the ES Sep/02 futures at the market.
-

Intraday Bars Exit Rule:

Close the position 1 minute before the E-Mini close (no trades will be carried overnight).

Walk Forward Optimization

Walk forward optimization will be used here because of the changing nature of the intraday E-Mini futures market. Intraday price dynamics are constantly changing due to current economic surprises, events and trader sentiment. Also the time of year changes the nature of intraday markets, such as the seasons, holidays, vacation time, etc. As such, optimizations on intraday data performed 3 months ago may no longer be representative of today's intraday price dynamics.

The walk forward procedure will be applied as follows. A test period of 4 weeks of the ES Sep/02 1 minute bar data, June 12th , 2002 through July 12th, 2002, is chosen and system parameter values are found through optimization on this intraday data segment. The parameter values found are then applied to the out-of-sample 1 minute intraday bar data following the test segment which in this case is July 15th, 2002 to July 19th, 2002.

Why a 4 week intraday data test segment? There is no correct ratio of test data needed to produce good one week intraday out-of-sample results. By experimenting with different window lengths, the four to one ratio seemed to work well. In walk forward testing, enough data is needed to model most of the price dynamics that will be encountered in the out-of-sample segment, but not so much data that when the price dynamics start to change they are swamped by the weight of distant past data price dynamics that no longer apply. It is important to get good results over time in walk forward testing because if you can not get good results in the out-of-sample segments, then the price dynamics cannot be modeled with the system . This means that real time performance will not match the optimized curve fitted performance results and be random using the model. Traders observe this type of random performance (that is it looks great on paper but falls apart in real time) when trying systems based on optimization curve fitting or anecdotal "proof" (looking at 3 or 4 successful cases only) without any out-of-sample testing .

Finding The System Parameters Using Walk Forward Optimization

There are three system parameters to find *thres, pup, and pdn*. The best parameters will be defined as those values that give the best Net Profits with the maximum winning bars, minimum losing bars, minimum drawdown, minimum largest losing trades. In addition, the results should be stable, e.g. the profits, wins, and drawdowns should not change by much as the parameters move by a small amount away from their optimum values. Also in choosing the "best" parameters, only those parameters sets whose maximum consecutive losses were 4 or less were considered. Optimization is defined as the search for the parameter values that give the best results as defined above. It should be noted that in this stage of system development, the only thing indicated by the optimum values that are found in the test portion is that the data has been

curve fitted as best it can with this system. Without further testing on out-of-sample data there is no way to tell if the system will work in the future.

It is not well known, but almost any real time series or even a random time series defined over a fixed number of bars can be curve fitted rather easily. The performance results and the statistical measurements that validate this performance of the curve fit will look excellent giving the false illusion of future profitability. However, the truth is that these excellent performance and associated statistics on the test section in no way validate how the system will perform on data it has not been optimized on. Only out-of-sample testing, that is testing on data the parameters were not derived on, can determine if the parameters found in the test section have captured the price dynamics. For instance in the End Point FFT process the error minimization forces the generated curve to fit the past data like a glove. It's almost impossible not to get an excellent fit with excellent statistical results. Unfortunately, this excellent fit in no way implies that the system will perform equally well on out-of-sample data, it just tells us we have a very good curve fit.

Results

Figure 4 presents a table of the test window optimum parameters for the S&P Sep/99 5min data series.

Start Date	End Date	Thres	Pntup	Pntdn
06/12/02	07/12/02	0.13	2.75	1.25

Figure 4 Optimum Parameter Values For Test Data Segment

Figures 5 presents the performance summary of the test segment using the optimum parameters shown in Figure 4.

Figure 6 presents the performance summary of the out-of-sample data segment from 07/15/02 to 07/19/02. This performance represents what would have happened in *real time* if one used the parameters found in the test section. Slippage, and commissions are not included.

Figure 7 presents a trade by trade summary from 06/12/02 to 07/19/02.

Figures 8A through 8E present the out-of-sample 1 minute bar charts of the E-Mini Sep/02 futures with the EPFFT Curve and all the buy and sell signals from the trade by trade summary of Figure 7 indicated on the charts.

Discussion of System Performance

As can be observed from the test sample Performance summary in Figure 5 and the out-of-sample performance summary of Figure 6, the out-of-sample "All Trades" performance was similar to the test sample "All Trades" performance. This similar performance in average winning and losing trades and drawdowns indicate that 4 weeks of test data was enough to capture the intraday price dynamics of the E-Mini Sep/02 contract for one week into the future.

Observing the out-of-sample trade by trade summary of Figure 7, we can see that the system did much better on the short trades than it did on the long trades. This better short performance is due to the fact that the E-Mini future dropped almost 8% in the out-of-sample week. However,

to the systems credit, the one big upside rally on 7/15/02 that started at 2:38 EST at 877.5 was caught be the system at 2:49 at 884.5 and exited at the top at 920.25 on the close.

In observing the charts, we can see that the system did very well in catching every major intraday trend . Overall the **EPFFT** system did a good job in minimizing the losses due to the inevitable whipsaws that will occur in any trading system and maximizing the profits from the major intraday trend moves of the E-Mini futures.

In order to use this system in real time trading, at least ten to twenty more test and out-of-sample windows from the past would have to be examined to gain confidence that the results above were not due to pure chance.

References:

Meyers, Dennis , "The Discrete Fourier Transform Illusion", *Stocks & Commodities*, Volume 17: April.1999

Meyers, Dennis , "The Endpoint Fast Fourier Transform System", *Stocks & Commodities*, Volume 17: May, 1999

Info on Dennis Meyers

Dennis Meyers (info@MeyersAnalytics.com.) has a doctorate in applied mathematics in engineering. He is a private trader, and president of Meyers Analytics (www.MeyersAnalytics.com). His firm specializes in Financial Engineering consulting for financial institutions and developing publicly available analytical software for traders.

Figure 5 Test Segment Performance Summary for E-Mini EPFFT System 06/12/02-07/12/02

EPFFT System- ES Sep/02 1 min 06/12/02-07/12/02

Performance Summary: All Trades

Total Net Profit	$8,525.00	Open position P/L	$0.00
Gross Profit	$17,450.00	Gross Loss	($8,925.00)
Total # of trades	95	Percent profitable	52.63%
Number winning trades	50	Number losing trades	45
Largest winning trade	$1,475.00	Largest losing trade	($625.00)
Average winning trade	$349.00	Average losing trade	($198.33)
Ratio avg win/avg los	1.76	Avg trade (win & los	$89.74
Max consec. Winners	6	Max consec. losers	4
Avg # bars in winners	120	Avg # bars in losers	47
Max intraday drawdown	($1,437.50)		
Profit Factor	1.96	Max # contracts held	1

Performance Summary: Long Trades

Total Net Profit	$2,700.00	Open position P/L	$0.00
Gross Profit	$6,837.50	Gross Loss	($4,137.50)
Total # of trades	43	Percent profitable	48.84%
Number winning trades	21	Number losing trades	22
Largest winning trade	$950.00	Largest losing trade	($375.00)
Average winning trade	$325.60	Average losing trade	($188.07)
Ratio avg win/avg los	1.73	Avg trade (win & los	$62.79
Max consec. Winners	5	Max consec. losers	8
Avg # bars in winners	71	Avg # bars in losers	34
Max intraday drawdown	($1,800.00)		
Profit Factor	1.65	Max # contracts held	1

Performance Summary: Short Trades

Total Net Profit	$5,825.00	Open position P/L	$0.00
Gross Profit	$10,612.50	Gross Loss	($4,787.50)
Total # of trades	52	Percent profitable	55.77%
Number winning trades	29	Number losing trades	23
Largest winning trade	$1,475.00	Largest losing trade	($625.00)
Average winning trade	$365.95	Average losing trade	($208.15)
Ratio avg win/avg los	1.76	Avg trade (win & los	$112.02
Max consec. Winners	4	Max consec. losers	4
Avg # bars in winners	154	Avg # bars in losers	61
Max intraday drawdown	($1,137.50)		
Profit Factor	2.22	Max # contracts held	1

Figure 6 Out-Of-Sample Performance Summary for E-Mini EPFFT System 06/12/02-07/12/02

EPFFT System- ES Sep/02 1 min 07/15/02-07/19/02

Performance Summary: All Trades

Total Net Profit	$3,962.50	Open position P/L	$0.00
Gross Profit	$6,900.00	Gross Loss	($2,937.50)
Total # of trades	36	Percent profitable	52.78%
Number winning trades	19	Number losing trades	17
Largest winning trade	$1,787.50	Largest losing trade	($487.50)
Average winning trade	$363.16	Average losing trade	($172.79)
Ratio avg win/avg los	2.10	Avg trade (win & los	$110.07
Max consec. Winners	5	Max consec. losers	4
Avg # bars in winners	73	Avg # bars in losers	36
Max intraday drawdown	($925.00)		
Profit Factor	2.35	Max # contracts held	1

Performance Summary: Long Trades

Total Net Profit	$425.00	Open position P/L	$0.00
Gross Profit	$2,212.50	Gross Loss	($1,787.50)
Total # of trades	17	Percent profitable	35.29%
Number winning trades	6	Number losing trades	11
Largest winning trade	$1,787.50	Largest losing trade	($487.50)
Average winning trade	$368.75	Average losing trade	($162.50)
Ratio avg win/avg los	2.27	Avg trade (win & los	$25.00
Max consec. Winners	3	Max consec. losers	6
Avg # bars in winners	35	Avg # bars in losers	29
Max intraday drawdown	($1,237.50)		
Profit Factor	1.24	Max # contracts held	1
Account size required	$1,237.50	Return on account	34.34%

Performance Summary: Short Trades

Total Net Profit	$3,537.50	Open position P/L	$0.00
Gross Profit	$4,687.50	Gross Loss	($1,150.00)
Total # of trades	19	Percent profitable	68.42%
Number winning trades	13	Number losing trades	6
Largest winning trade	$1,350.00	Largest losing trade	($350.00)
Average winning trade	$360.58	Average losing trade	($191.67)
Ratio avg win/avg los	1.88	Avg trade (win & los	$186.18
Max consec. Winners	4	Max consec. losers	2
Avg # bars in winners	91	Avg # bars in losers	49
Max intraday drawdown	($612.50)		
Profit Factor	4.08	Max # contracts held	1
Account size required	$612.50	Return on account	577.55%

FIGURE 7 Test & Out-Of-Sample Trade By Trade Summary
E-Mini 1min Bars EPFFT System 06/13/02-07/19/02

Entry Date	Entry Time		Entry Price	Exit Time	Exit Price	Bars InTrd	Trade $P&L	Trade Max$Pft	Time	Trade Max$DD	Time
6/13/02	1200	Buy	1023.5	1249	1020.25	49	($162.50)	$75.00	1227	($162.50)	1247
6/13/02	1249	Sell	1020.25	1614	1011.75	205	$425.00	$550.00	1555	($137.50)	1345
6/14/02	932	Sell	998.75	1032	993	60	$287.50	$887.50	1001	$0.00	932
6/14/02	1032	Buy	993	1218	1002.75	106	$487.50	$587.50	1141	($100.00)	1033
6/14/02	1218	Sell	1002.75	1410	1001.5	112	$62.50	$350.00	1246	($62.50)	1221
6/14/02	1410	Buy	1001.5	1511	1004.25	61	$137.50	$437.50	1446	$0.00	1410
6/14/02	1511	Sell	1004.25	1550	1007	39	($137.50)	$62.50	1517	($275.00)	1539
6/14/02	1550	Buy	1007	1614	1009.75	24	$137.50	$150.00	1612	($25.00)	1552
6/17/02	932	Buy	1015.75	1215	1032	163	$812.50	$900.00	1155	($37.50)	932
6/17/02	1215	Sell	1032	1614	1036.5	239	($225.00)	$150.00	1327	($325.00)	1605
6/18/02	932	Sell	1032.25	1614	1044.75	402	($625.00)	$0.00	932	($637.50)	1613
6/19/02	933	Sell	1030.25	1108	1032.25	95	($100.00)	$137.50	953	($150.00)	937
6/19/02	1108	Buy	1032.25	1203	1036.75	55	$225.00	$325.00	1152	$0.00	1108
6/19/02	1203	Sell	1036.75	1614	1020.25	251	$825.00	$912.50	1533	($25.00)	1204
6/20/02	932	Sell	1018.5	1330	1016.25	238	$112.50	$387.50	1239	($325.00)	1002
6/20/02	1330	Buy	1016.25	1519	1011.5	109	($237.50)	$75.00	1447	($287.50)	1516
6/20/02	1519	Sell	1011.5	1614	1007.75	55	$187.50	$312.50	1541	($12.50)	1520
6/21/02	932	Sell	999.5	1010	1007	38	($375.00)	$62.50	939	($375.00)	1009
6/21/02	1010	Buy	1007	1033	1003	23	($200.00)	$0.00	1010	($200.00)	1033
6/21/02	1033	Sell	1003	1404	996.75	211	$312.50	$550.00	1246	($25.00)	1051
6/21/02	1404	Buy	996.75	1523	991.5	79	($262.50)	$12.50	1505	($337.50)	1520
6/21/02	1523	Sell	991.5	1607	991.25	44	$12.50	$300.00	1551	($37.50)	1530
6/21/02	1607	Buy	991.25	1614	991.25	7	$0.00	$25.00	1608	$0.00	1607
6/24/02	935	Sell	989.25	1312	978.5	217	$537.50	$912.50	1252	($125.00)	941
6/24/02	1312	Buy	978.5	1529	995	137	$825.00	$1,300.00	1436	($25.00)	1314
6/24/02	1529	Sell	995	1614	996.25	45	($62.50)	$300.00	1542	($62.50)	1607
6/25/02	932	Sell	1000.25	942	1001.5	10	($62.50)	$0.00	932	($112.50)	937
6/25/02	942	Buy	1001.5	1104	1004.25	82	$137.50	$287.50	1038	($200.00)	1011
6/25/02	1104	Sell	1004.25	1614	974.75	310	$1,475.00	$1,512.50	1546	($87.50)	1105
6/26/02	932	Sell	952.5	1020	959	48	($325.00)	$50.00	933	($575.00)	1001
6/26/02	1020	Buy	959	1057	957.5	37	($75.00)	$275.00	1033	($100.00)	1054
6/26/02	1057	Sell	957.5	1131	960.75	34	($162.50)	$0.00	1057	($312.50)	1114
6/26/02	1131	Buy	960.75	1230	965.5	59	$237.50	$600.00	1200	$0.00	1131
6/26/02	1230	Sell	965.5	1452	963.75	142	$87.50	$550.00	1432	($137.50)	1236
6/26/02	1452	Buy	963.75	1542	972.5	50	$437.50	$775.00	1528	($137.50)	1514
6/26/02	1542	Sell	972.5	1603	973.5	21	($50.00)	$187.50	1549	($150.00)	1559
6/26/02	1603	Buy	973.5	1614	974.5	11	$50.00	$87.50	1603	$0.00	1603
6/27/02	932	Buy	983	1029	978	57	($250.00)	$37.50	938	($287.50)	954
6/27/02	1029	Sell	978	1304	974	155	$200.00	$675.00	1124	($62.50)	1049
6/27/02	1304	Buy	974	1509	984.5	125	$525.00	$750.00	1452	($75.00)	1307
6/27/02	1509	Sell	984.5	1545	987.5	36	($150.00)	$237.50	1525	($162.50)	1542
6/27/02	1545	Buy	987.5	1614	991.5	29	$200.00	$262.50	1603	($37.50)	1548
6/28/02	932	Buy	990.5	1037	1000.25	65	$487.50	$612.50	1021	($75.00)	932
6/28/02	1037	Sell	1000.25	1614	987.5	337	$637.50	$637.50	1612	($112.50)	1411
7/1/02	932	Sell	989.5	954	994.5	22	($250.00)	$0.00	932	($300.00)	952
7/1/02	954	Buy	994.5	1013	987.5	19	($350.00)	$0.00	954	($362.50)	1011
7/1/02	1013	Sell	987.5	1329	984.5	196	$150.00	$437.50	1311	($187.50)	1056
7/1/02	1329	Buy	984.5	1353	982.5	24	($100.00)	$50.00	1334	($112.50)	1352
7/1/02	1353	Sell	982.5	1614	968	141	$725.00	$762.50	1613	($162.50)	1418
7/2/02	932	Sell	966.75	1334	951.5	242	$762.50	$1,037.50	1316	($137.50)	956
7/2/02	1334	Buy	951.5	1540	950.5	126	($50.00)	$400.00	1409	($187.50)	1533
7/2/02	1540	Sell	950.5	1614	948	34	$125.00	$125.00	1612	($125.00)	1546

7/3/02	932	Sell	945.75	958	952.5	26	($337.50)	$75.00	934	($337.50)	951
7/3/02	958	Buy	952.5	1023	945	25	($375.00)	$37.50	1001	($375.00)	1020
7/3/02	1023	Sell	945	1152	942.5	89	$125.00	$475.00	1128	($37.50)	1023
7/3/02	1152	Buy	942.5	1219	941	24	($75.00)	$162.50	1205	($125.00)	1217
7/3/02	1219	Sell	941	1307	942.75	48	($87.50)	$212.50	1235	($87.50)	1306
7/3/02	1307	Buy	942.75	1614	953.5	187	$537.50	$625.00	1559	($200.00)	1320
7/5/02	932	Buy	965.25	1225	984.25	173	$950.00	$1,025.00	1156	$0.00	932
7/5/02	1225	Sell	984.25	1314	992.5	49	($412.50)	$50.00	1235	($425.00)	1309
7/5/02	1314	Buy	992.5	1315	991	1	($75.00)	$0.00	1314	($75.00)	1315
7/8/02	931	Buy	987.5	949	986.75	18	($37.50)	$325.00	939	($37.50)	948
7/8/02	949	Sell	986.75	1614	977.5	385	$462.50	$650.00	1538	($100.00)	952
7/9/02	932	Sell	975.5	1043	977.5	71	($100.00)	$212.50	1014	($212.50)	942
7/9/02	1043	Buy	977.5	1057	974.75	14	($137.50)	$12.50	1045	($175.00)	1055
7/9/02	1057	Sell	974.75	1614	953.5	317	$1,062.50	$1,150.00	1555	($287.50)	1125
7/10/02	932	Sell	956.25	1015	955.75	43	$25.00	$500.00	952	($12.50)	932
7/10/02	1015	Buy	955.75	1033	950.75	18	($250.00)	$12.50	1016	($250.00)	1033
7/10/02	1033	Sell	950.75	1102	951	29	($12.50)	$150.00	1035	($100.00)	1056
7/10/02	1102	Buy	951	1114	944.5	12	($325.00)	$12.50	1102	($325.00)	1114
7/10/02	1114	Sell	944.5	1254	941.25	100	$162.50	$487.50	1240	($87.50)	1118
7/10/02	1254	Buy	941.25	1318	940.5	24	($37.50)	$125.00	1303	($75.00)	1314
7/10/02	1318	Sell	940.5	1457	931.25	99	$462.50	$837.50	1431	($25.00)	1323
7/10/02	1457	Buy	931.25	1511	924.25	14	($350.00)	$37.50	1457	($350.00)	1511
7/10/02	1511	Sell	924.25	1535	926	24	($87.50)	$187.50	1522	($87.50)	1534
7/10/02	1535	Buy	926	1546	920.75	11	($262.50)	$87.50	1538	($262.50)	1545
7/10/02	1546	Sell	920.75	1614	920	28	$37.50	$137.50	1609	($62.50)	1552
7/11/02	932	Sell	915	958	916.5	26	($75.00)	$400.00	947	($75.00)	957
7/11/02	958	Buy	916.5	1013	917	15	$25.00	$387.50	1005	($37.50)	959
7/11/02	1013	Sell	917	1030	923.5	17	($325.00)	$0.00	1013	($362.50)	1027
7/11/02	1030	Buy	923.5	1037	918.75	7	($237.50)	$0.00	1030	($250.00)	1035
7/11/02	1037	Sell	918.75	1243	906.75	126	$600.00	$887.50	1217	($137.50)	1043
7/11/02	1243	Buy	906.75	1313	909.75	30	$150.00	$362.50	1259	($100.00)	1245
7/11/02	1313	Sell	909.75	1332	912	19	($112.50)	$125.00	1315	($125.00)	1329
7/11/02	1332	Buy	912	1424	918.25	52	$312.50	$650.00	1411	$0.00	1332
7/11/02	1424	Sell	918.25	1443	923	19	($237.50)	$75.00	1424	($337.50)	1439
7/11/02	1443	Buy	923	1457	918	14	($250.00)	$25.00	1443	($262.50)	1455
7/11/02	1457	Sell	918	1534	927.5	37	($475.00)	$125.00	1501	($475.00)	1533
7/11/02	1534	Buy	927.5	1607	926.75	33	($37.50)	$137.50	1540	($37.50)	1607
7/11/02	1607	Sell	926.75	1614	926.75	7	$0.00	$0.00	1607	($50.00)	1611
7/12/02	932	Sell	930.25	1029	929.75	57	$25.00	$825.00	956	($87.50)	934
7/12/02	1029	Buy	929.75	1110	932	41	$112.50	$262.50	1058	($100.00)	1030
7/12/02	1110	Sell	932	1542	920.25	272	$587.50	$925.00	1452	($87.50)	1112
7/12/02	1542	Buy	920.25	1609	921.25	27	$50.00	$187.50	1550	($37.50)	1559
7/12/02	1609	Sell	921.25	1614	918.5	5	$137.50	$137.50	1614	$0.00	1609
Out-Of_Sample Trades Below											
7/15/02	932	Sell	912.25	1056	905.5	84	$337.50	$637.50	1015	$0.00	932
7/15/02	1056	Buy	905.5	1116	899.5	20	($300.00)	$0.00	1056	($325.00)	1112
7/15/02	1116	Sell	899.5	1242	894	86	$275.00	$487.50	1220	($37.50)	1156
7/15/02	1242	Buy	894	1308	891.5	26	($125.00)	$87.50	1244	($137.50)	1307
7/15/02	1308	Sell	891.5	1449	884.5	101	$350.00	$750.00	1437	($137.50)	1324
7/15/02	1449	Buy	884.5	1614	920.25	85	$1,787.50	$1,825.00	1613	$0.00	1449
7/16/02	932	Buy	909	939	913	7	$200.00	$200.00	937	($12.50)	934
7/16/02	939	Sell	913	1100	907.75	81	$262.50	$812.50	1033	($87.50)	939
7/16/02	1100	Buy	907.75	1132	909	32	$62.50	$525.00	1108	$0.00	1100
7/16/02	1132	Sell	909	1302	916	90	($350.00)	$50.00	1144	($500.00)	1258
7/16/02	1302	Buy	916	1320	915.5	18	($25.00)	$187.50	1309	($37.50)	1303
7/16/02	1320	Sell	915.5	1441	916	81	($25.00)	$337.50	1412	($150.00)	1343
7/16/02	1441	Buy	916	1502	912.75	21	($162.50)	$12.50	1457	($187.50)	1445

7/16/02	1502	Sell	912.75	1614	901.5	72	$562.50	$737.50	1536	$0.00	1502
7/17/02	932	Sell	923	936	925.25	4	($112.50)	$25.00	934	($112.50)	935
7/17/02	936	Buy	925.25	1000	920.75	24	($225.00)	$162.50	940	($275.00)	956
7/17/02	1000	Sell	920.75	1139	913	99	$387.50	$712.50	1129	($237.50)	1011
7/17/02	1139	Buy	913	1149	910.25	10	($137.50)	$37.50	1139	($137.50)	1148
7/17/02	1149	Sell	910.25	1320	901.75	91	$425.00	$737.50	1301	($150.00)	1155
7/17/02	1320	Buy	901.75	1343	900.5	23	($62.50)	$137.50	1333	($112.50)	1324
7/17/02	1343	Sell	900.5	1402	906.5	19	($300.00)	$25.00	1343	($312.50)	1358
7/17/02	1402	Buy	906.5	1413	905	11	($75.00)	$112.50	1403	($87.50)	1412
7/17/02	1413	Sell	905	1513	902.5	60	$125.00	$375.00	1454	($162.50)	1422
7/17/02	1513	Buy	902.5	1522	902.5	9	$0.00	$0.00	1513	($150.00)	1517
7/17/02	1522	Sell	902.5	1536	907	14	($225.00)	$0.00	1522	($375.00)	1533
7/17/02	1536	Buy	907	1544	908.25	8	$62.50	$62.50	1543	($125.00)	1539
7/17/02	1544	Sell	908.25	1606	906.75	22	$75.00	$250.00	1552	$0.00	1544
7/17/02	1606	Buy	906.75	1614	904.5	8	($112.50)	$0.00	1606	($112.50)	1607
7/18/02	932	Sell	903	1117	902	105	$50.00	$387.50	1002	($187.50)	938
7/18/02	1117	Buy	902	1228	904	71	$100.00	$250.00	1200	($100.00)	1204
7/18/02	1228	Sell	904	1614	877	226	$1,350.00	$1,437.50	1603	$0.00	1228
7/19/02	932	Sell	864.5	1058	867.25	86	($137.50)	$150.00	938	($362.50)	1016
7/19/02	1058	Buy	867.25	1113	865.75	15	($75.00)	$0.00	1058	($187.50)	1102
7/19/02	1113	Sell	865.75	1217	863.25	64	$125.00	$387.50	1155	$0.00	1113
7/19/02	1217	Buy	863.25	1445	853.5	148	($487.50)	$25.00	1343	($500.00)	1443
7/19/02	1445	Sell	853.5	1614	846.25	89	$362.50	$650.00	1553	($337.50)	1455

Figure 1 Noise Filtered FFT on SP Daily 1/16/1998 to 1/22/1999

Figure 2 Noise Filtered FFT on SP Daily 7/15/1997 to 7/20/1998

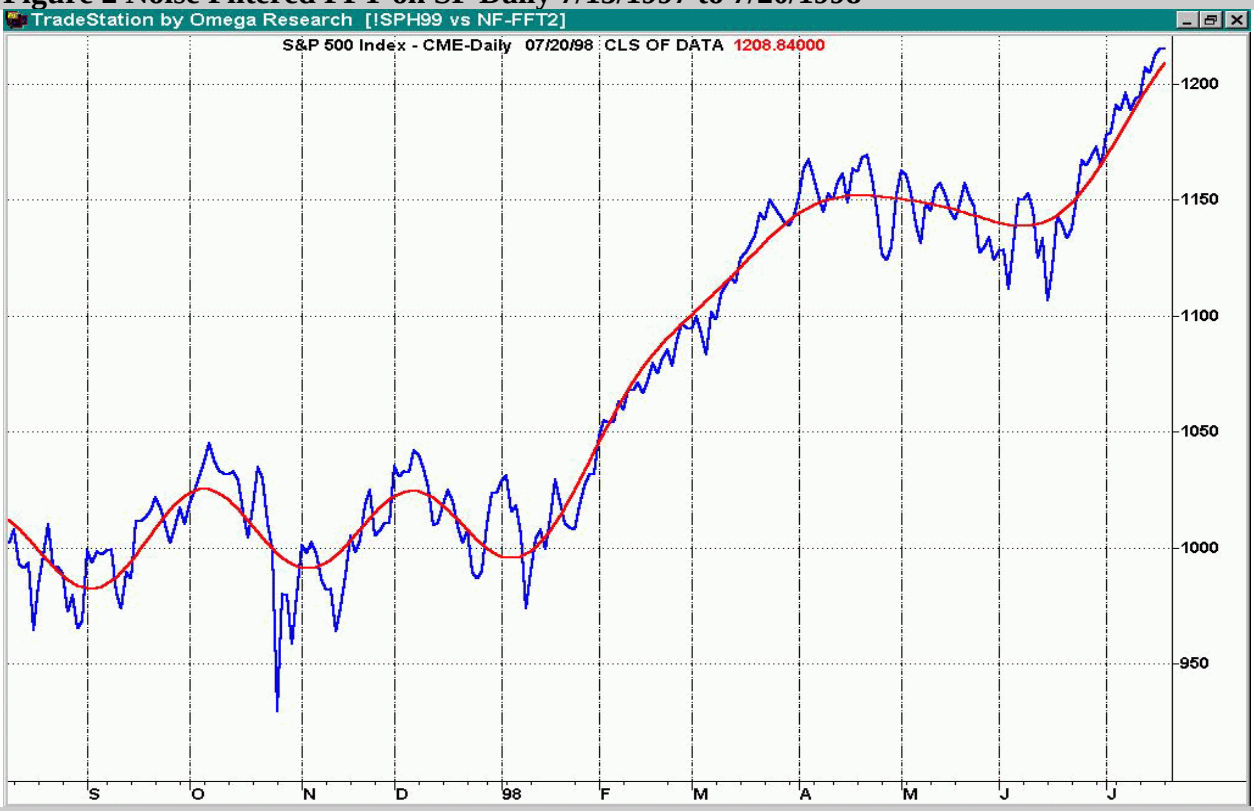
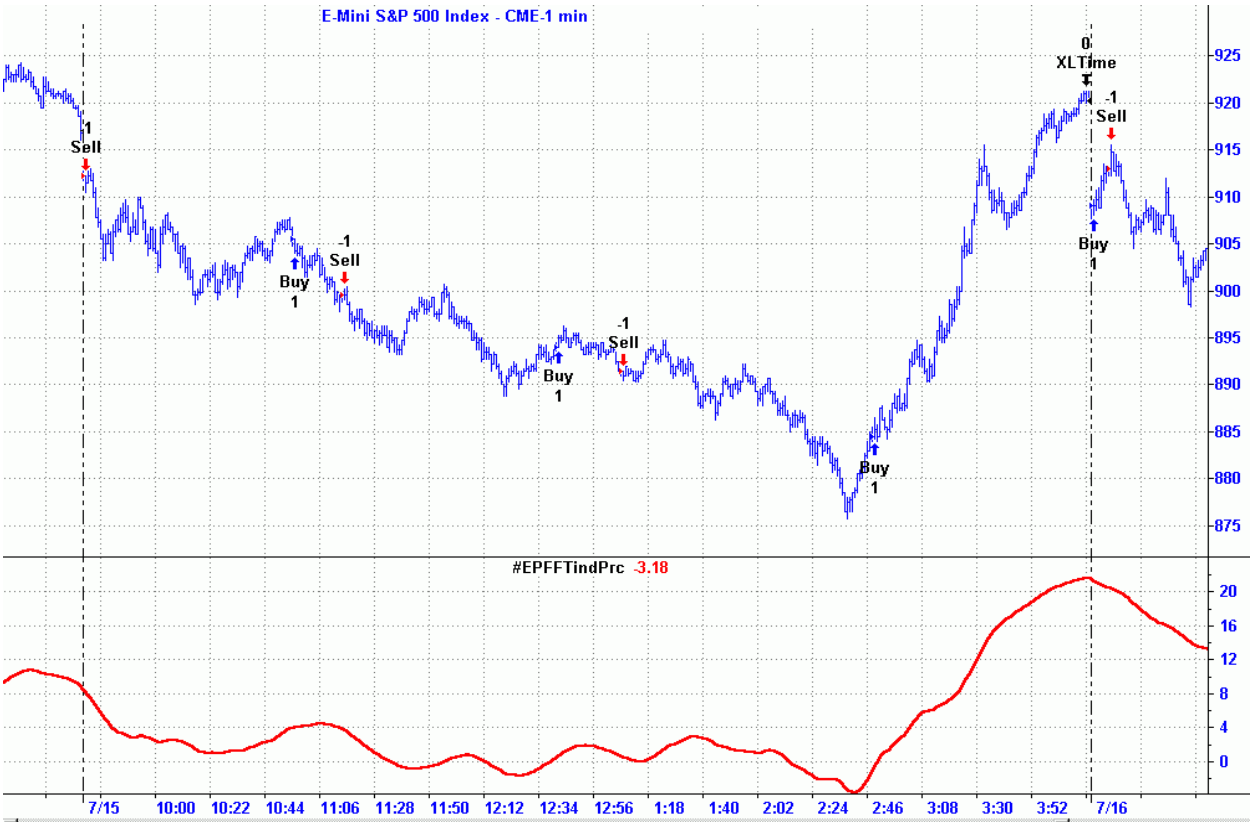


Figure 3 Noise Filtered EPFFT on SP Daily 2/15/1998 to 1/15/1999

Figure 8 Out-Of-Sample Chart for E-Mini 1min Bars EPFFT System 7/15/02-07/19/02



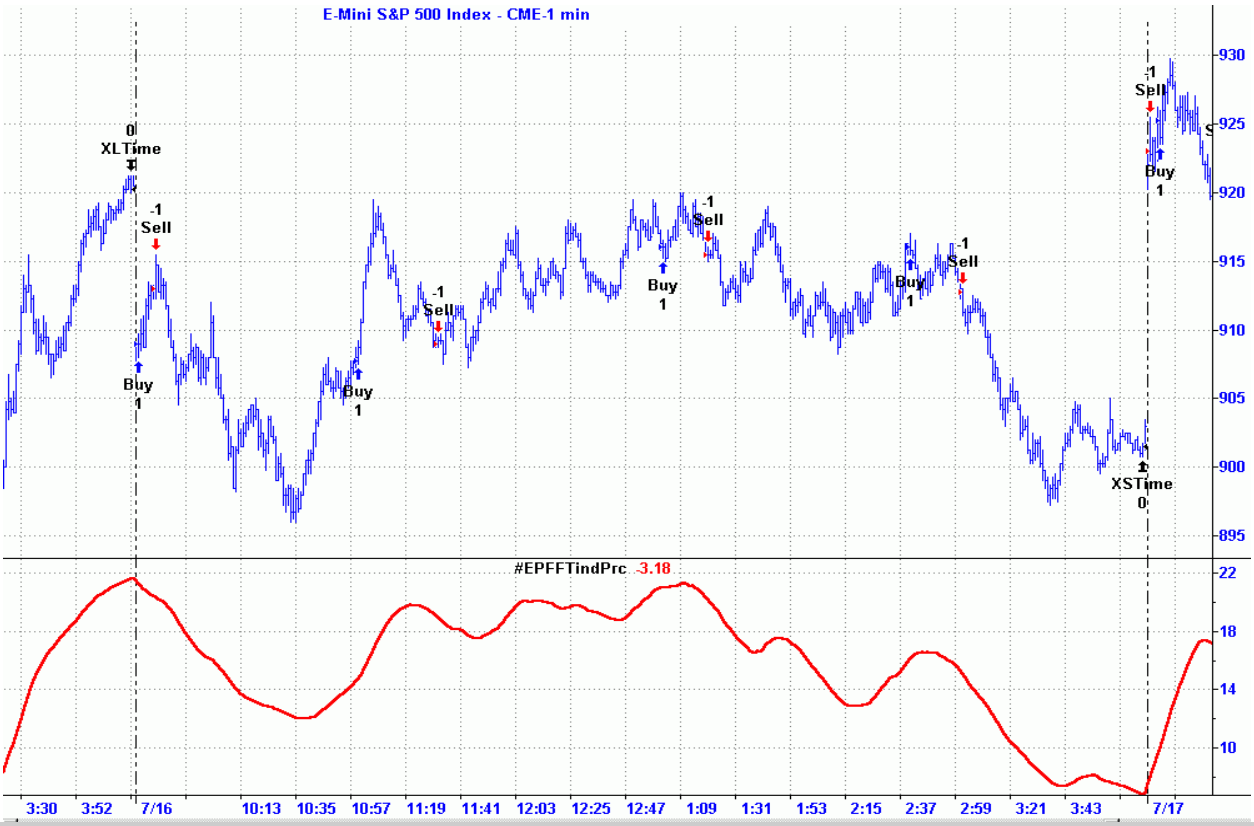
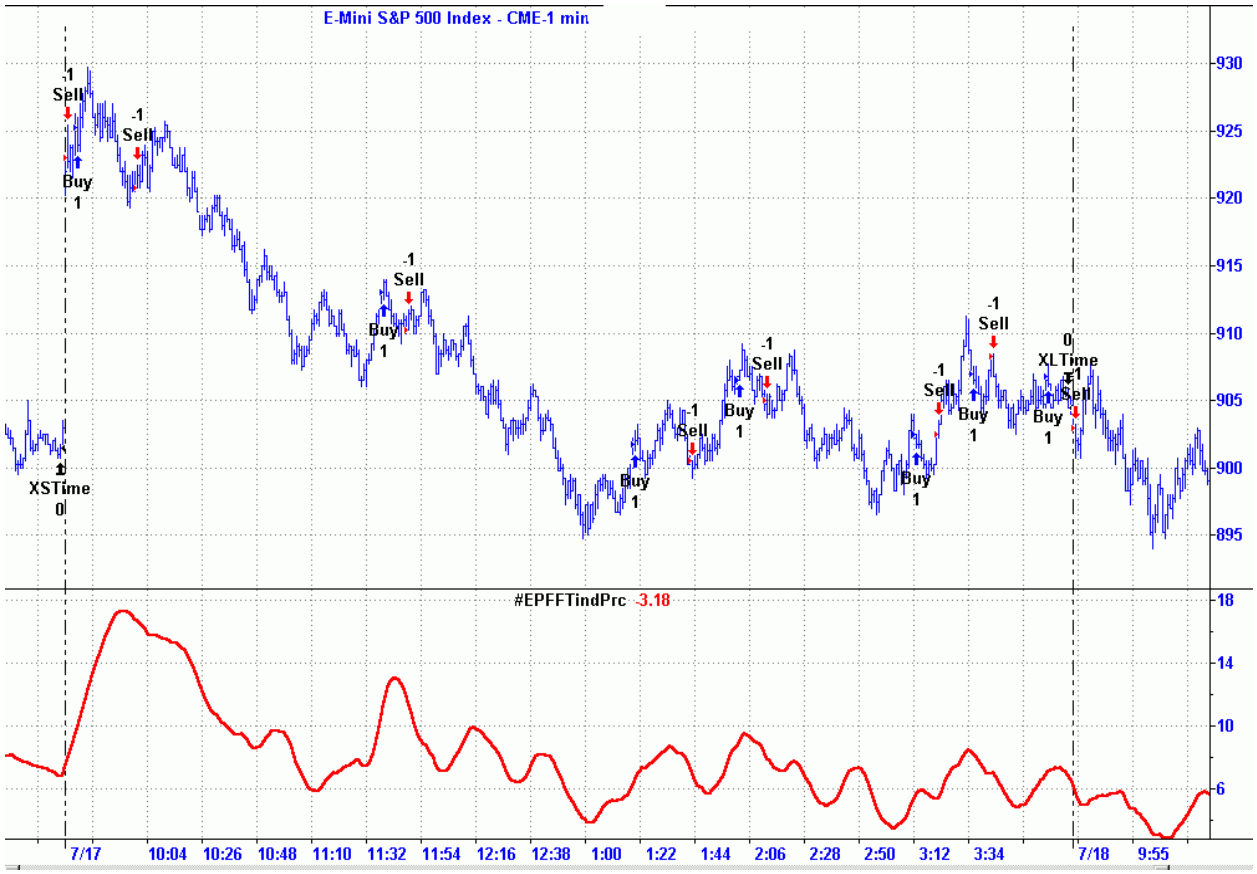


Figure 8 Out-Of-Sample Chart for E-Mini 1min Bars EPFFT System 7/15/02-07/19/02



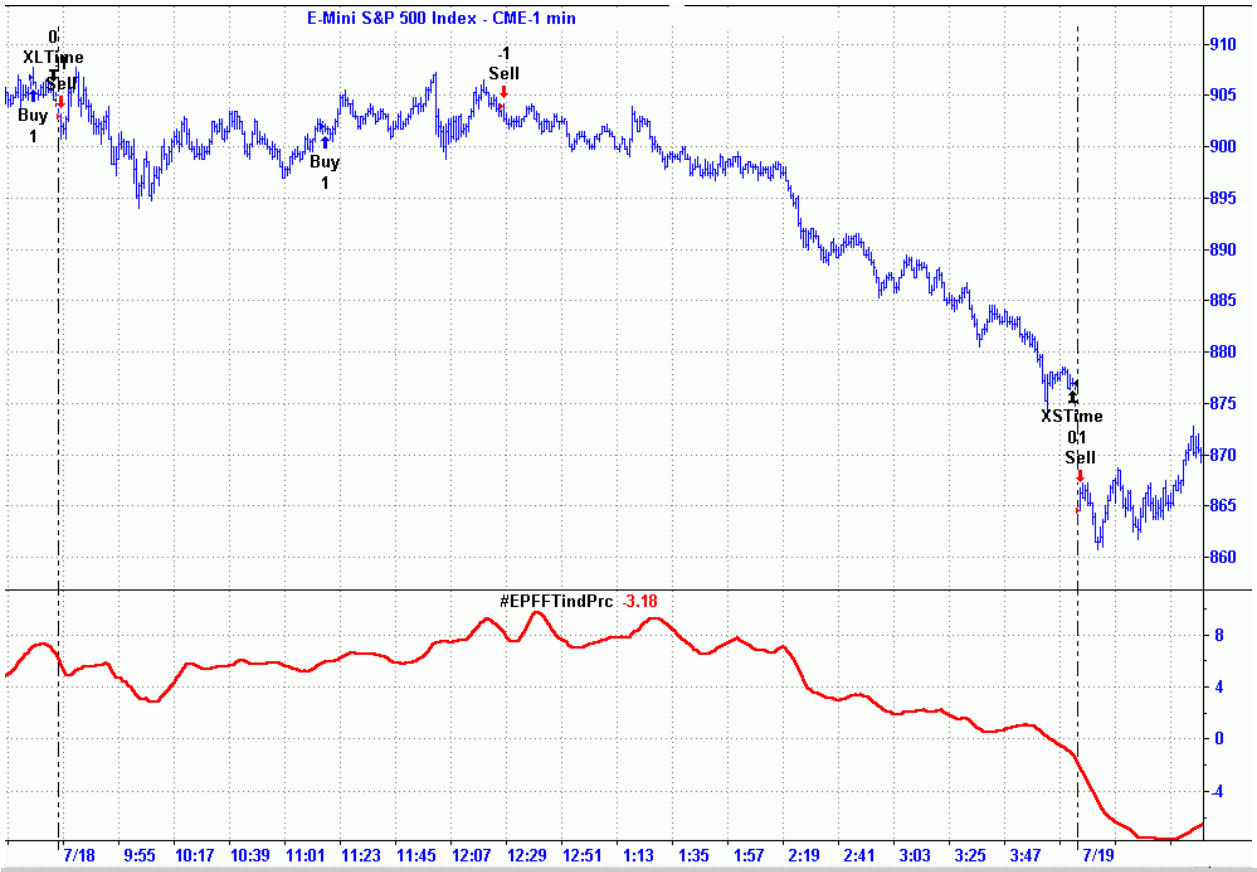
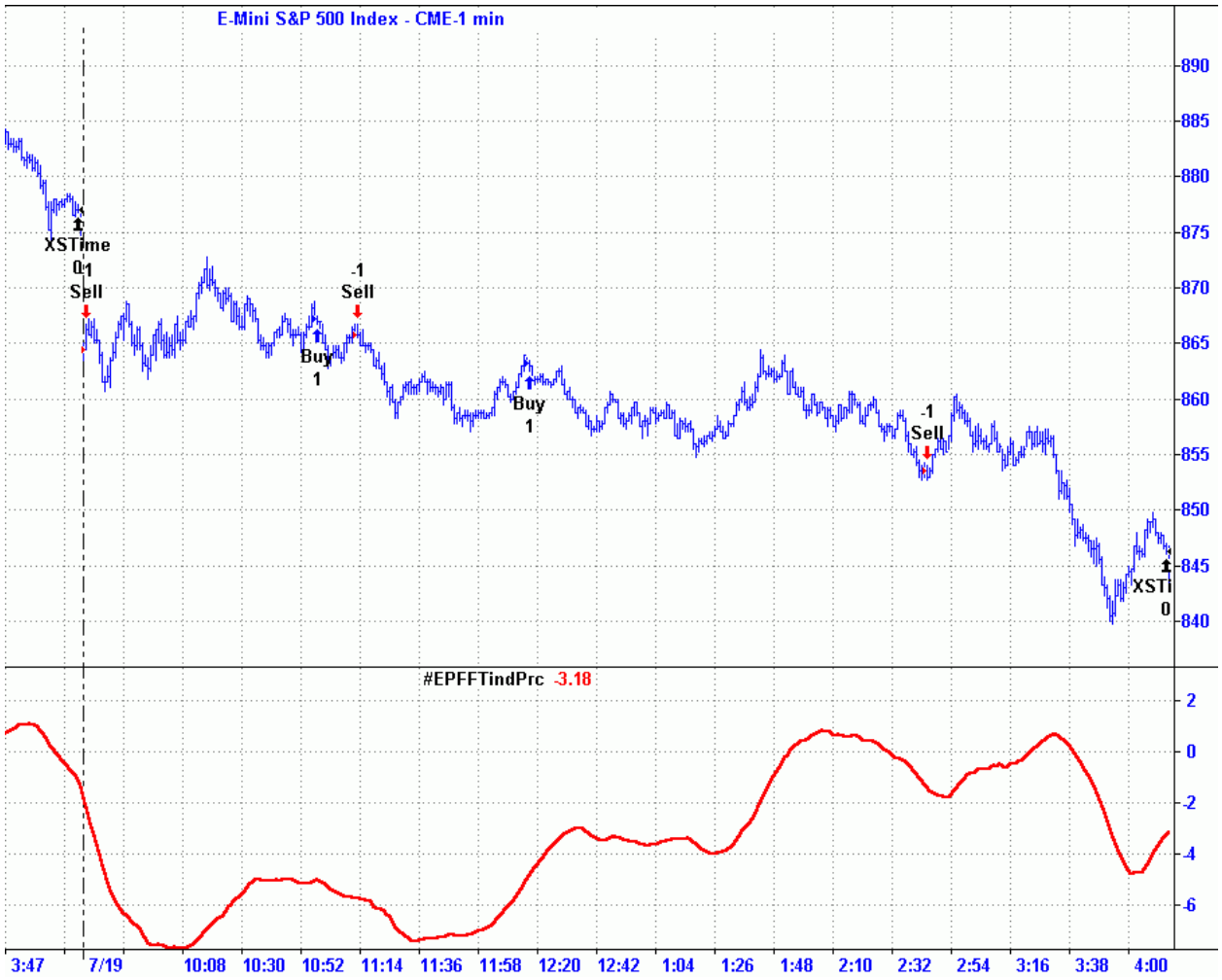



Figure 8 Out-Of-Sample Chart for E-Mini 1min Bars EPFFT System 7/15/02-07/19/02