

Trading ES 5 Min Bars With The Least Squares Velocity Strategy With Walk Forward 4 days in-sample and 1 day out-of-sample

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In previous working papers [Ref below] we examined a trading system that used the velocity of prices fit by a least squares straight line through “N” past prices, to determine buy and sell points. The reasoning behind this type of system was to only trade when the straight line slope or velocity was above a certain threshold. Many times during the day prices meandered around without a notable trend. At these times we do not wish to trade because of the whipsaw losses that occur from this type of price action. When a price trend finally starts, the velocity of that price trend moves above some minimum threshold value. Thus the velocity system would only issue a trade when certain velocity barriers were crossed.

The Least Squares polynomial is determined by minimizing the sum of the squares of the difference between the N prices and the value of the polynomial line.

$$\text{err}^2(t) = [\text{Price}(t) - (a + b * t)]^2 = \text{error squared}$$

$$\text{Minimize}(a, b) \sum_{t=1}^{t=N} \text{err}^2(t)$$

This mathematical technique has an exact solution and dates back to Gauss in the 1800's.

The Least Squares Velocity

Let us imagine a set of closing prices on a graph with time as the horizontal axis and price as the vertical axis. Let us further suppose that we have only twenty closing price dots at twenty time intervals. How can we draw a straight line through those twenty prices such that the sum of all the squared differences between the prices at each time interval and the straight line that is being fit to the data is minimized? This is called the “Least Squares Fit” line of the data (also referred to as the linear regression line). This mathematical technique is available in most of today's technical analysis software.

The formula for the straight line is:

$$y = a + b * t$$

where **a** is the initial value of the line, **b** is the slope of the line, and **t** is the time of the bar. The slope **b** is also called the **velocity**. Recall that velocity is defined as the change of position per unit time. Using the formula above as an easy way to visualize dy/dt , the derivative of y with respect to t , the velocity would be:

$$\text{Velocity} = [a + b * (t + 1)] - [a + b * t] = b$$

If you are fitting the straight line to N prices then the “Best Fit” coefficients **a** and **b** can be solved for quite easily and are given by

$$a = [2(2N+1)/N(N-1)] \sum_1^N p(t) - [6/(N(N-1))] \sum_1^N t * p(t)$$

$$b = \text{Velocity} = [12/N(N^2 - 1)] \sum_1^N t * p(t) - [6/N(N-1)] \sum_1^N p(t)$$

Where $p(t)$ is the price at point t and N is the number of prices we are using to calculate the coefficients. Here $p(1)$ is the first price in the series and $p(N)$ is the last price in the series.

Here we will use the **velocity** of the least squares straight line to create a strategy. The least squares velocity has the advantage that it is a natural random price noise inhibitor. We can create a strategy such that unless the velocity is greater than some threshold we will not buy or sell. A large percentage of price noise generates a lot of back and forth movements of small magnitudes. With a lot of strategies this back and forth movement creates many false buy and sell signals. However using the least squares velocity we can filter many of the small price noise movements by requiring that the velocity be greater than some threshold before we act.

The Least Squares Velocity Strategy Defined

At each bar we calculate the least squares **velocity** or **b** from the formula above. When the velocity is greater than the threshold amount **vup** we will go long. When the velocity is less than the threshold amount **-vdn** we will go short.

Buy Rule:

IF Velocity is greater than the threshold amount **vup** then buy at the market.

Sell Rule:

IF Velocity is less than the threshold amount **-vdn** then sell at the market.

Intraday Bars Exit Rule:

Close all positions 15 minutes before the ES close (1500) (no trades will be carried overnight).

Discussion of S&P500 Index E-Mini Future Prices

The **S&P 500 Index E-Mini Future (ES)** is traded on the CME Futures Globex Exchange and is traded on a 23 hour basis. We have restricted our study to only trading the ES during the stock market hours of 8:30 to 1500 CST. To test this strategy we will use 5 minute bar prices of the ES futures contract from August 1, 2014 to October 30, 2015

Testing The Least Squares Velocity Strategy(LSqV) Using Walk Forward Optimization

There are three strategy inputs to determine:

1. N , is the lookback period to calculate the **LSqV**.
2. **vup**, the threshold amount that LSqV has to be greater than to issue a buy signal
3. **vdn**, the threshold amount that LSqV has to be less than to issue a sell signal

We will test the LSqV strategy with the above ES 5 min bars on a **walk forward basis**, where the in-sample(IS) will be 4 weekdays and the out-of-sample(OOS) will be the next trading weekday following. as will be described below.

What Is A Walk Forward Optimization with In-Sample Section and Out-Of-Sample Sections?

Whenever we do a TS optimization on a number of different strategy inputs, TS generates a report of performance metrics (total net profits, number of losing trades, etc) vs these different strategy inputs. If the report is sorted on say the total net profits(*tnp*) performance metric column then the highest *tnp* would correspond to a certain set of inputs. This is called a *in-sample section*. If we choose a set of strategy inputs from this report based upon some performance metric, we have no idea whether these strategy inputs will produce the same results on future price data or data they have not been tested on. Price data that is not in the in-sample section is defined as *out-of-sample data*. Since the performance metrics generated in the in-sample section are mostly due to “curve fitting” or “data mining” it is important to see how the strategy inputs chosen from the in-sample section perform on out-of-sample data.

What do we mean by “*curve fitting*” or *data mining*? As a simple example, suppose you were taking a subway to work. In the subway car you are in, suppose you counted the number of blond women in that car and suppose the percent of blond women vs all other women hair colors was 80%. Being that you can't observe what is in the other subway cars, you would assume that all the other subway cars and perhaps all women had the same percentage of blond hair. This observation was due to chance. That is an example of curve fitting. The same goes for combinatorial searches. You are observing results from a finite sample of data without knowing the data outside the sample you examined.

Walk forward analysis attempts to minimize the curve fitting of price noise by using the law of averages from the Central Limit Theorem on the out-of-sample performance. In walk forward analysis the data is broken up into many in-sample and out-of-sample sections. Usually for any strategy, one has some performance metric selection procedure, which we will call a *filter*, used to select the input parameters from the optimization run. For instance, a *filter* example might be all cases that have a profit factor (PF) greater than 1 and less than 3. For the number of cases left, we might select the cases that had the best percent profit. This procedure would leave you with one case in the in-sample section output and its associated strategy input parameters. Now suppose we ran our optimization on each of our many in-sample sections and applied our filter to each in-sample section output. We would then use the strategy input parameters found by the *filter* in each in-sample section on the out-of-sample section immediately following that in-sample section. The input parameters found in each in-sample section and applied to each out-of-sample section would produce independent net profits or losses for each of the out-of-sample sections. Using this method we now have "x" number of independent out-of-sample section profit and losses from our filter. If we take the average of these out-of-sample section net profits and losses, then we will have an estimate of how our strategy will perform on average. Due to the Central Limit Theorem, as your sample size increases, the spurious noise results in the out-of-sample section performance tend to average out to zero in the limit, leaving us with what to expect from our strategy and filter. *Mathematical note: This assumption assumes that the out-of-sample returns are from probability distributions that have a finite variance.*

Why use the walk forward technique? Why not just perform an optimization on the whole price series and choose the input parameters that give the best total net profits or profit factor? Surely the price noise cancels itself out with such a large number of in-sample prices and trades. Unfortunately, nothing could be farther from the truth! Optimization is a misnomer and should

really be called combinatorial search. As stated above, whenever we run a combinatorial search over many different combinations of input parameters on noisy data on a fixed number of prices, **no matter how many**, the best performance parameters found are guaranteed to be due to “**curve fitting**” the noise and signal. The price series that we trade consists of random spurious price movements, which we call noise, and repeatable price patterns (*if they exist*). When we run, for example, 5000 different inputs parameter combinations, the best performance parameters will be from those strategy input variables that are able to produce profits from the price pattern **and** the random spurious movements. While the price patterns will repeat, the same spurious price movements will not. If the spurious price movements that were captured by a certain set of input parameters were a large part of the total net profits, as they are in real intraday price series, then choosing these input parameters will produce losses when traded on future data. These losses occur because the spurious price movements will not be repeated in the same way. This is why strategy optimization or combinatorial searches with no out-of-sample testing cause losses when traded in real time from something that looked great in the in-sample section.

In order to gain confidence that our input parameter selection method using the optimization output of the in-sample data will produce profits, we must test the input parameters we found in the in-sample section on out-of-sample data. In addition, we must perform the in-sample/out-of-sample analysis many times. Why not just do the out-of-sample analysis once or just 10 times? Well just as in Poker or any card game, where there is considerable variation in luck from hand to hand, walk forward out-of-sample analysis give considerable variation in day-to-day out-of-sample profit “luck”. That is, by pure chance we may have chosen some input parameter set that did well in the in-sample section data **and** the out-of-sample section data. In order to minimize this type of “luck”, statistically, we must repeat the walk forward out-of-sample (oos) analysis over many (>50) in-sample/out-of-sample sections and take an average over all out-of-sample sections. This average gives us an expected out-of-sample return and a standard deviation of out-of-sample returns which allows us to statistically estimate the expected equity and its range for N out-of-sample periods in the future

Finding The Strategy Parameters Using Walk Forward Optimization

There are three strategy parameters to find *N*, *vup* and *vdn*.

For the test data we will run the TradeStation optimization engine on ES 5 min price bars from 8/1/2014 to 10/30/2015 with the following optimization ranges for the Least squares velocity strategy inputs. This will create **308 4 weekday in-sample periods each followed by a 1 day out-of-sample period** (See Figure 1 for the in-sample/out-of-sample periods). The days are weekdays only. Weekdays where the OOS falls on a weekday exchange holiday or partial holidays are eliminated. Holidays that fall on a weekday create a 3day IS. All other **IS** periods consist of 4 trading weekdays.

The optimization ranges are:

1. N from 4 to 16 in steps of 2
2. vup from 0.25 to 3.5 in steps of 0.25
3. vdn from 0.25 to 3.5 in steps of 0.25
4. Mult=0.5359* \sqrt{N} Note: this normalizes the Velocity standard deviation for each different lookback period, N, to one standard deviation. Else the Velocity would have different ranges for different N and it would be difficult to find a vup and vdn that worked for all N ranges. See Appendix for a detailed explanation.

The above N, vup and vdn ranges will produce 1176 different input combinations or cases of the strategy input parameters for each of the 308 in-sample/out-of-sample files for the 15 months of 1 min bar ES data.

The question we are attempting to answer statistically is which best performance metric or combination of best performance metrics (which we will call a *filter*) applied to the in-sample section will produce in-sample strategy inputs that produce statistically valid profits in the out-of-sample section. In other words we wish to find a performance metric *filter* that we can apply to the in-sample section that can give us strategy inputs that will produce, on average, good trading results in the future.

When TS does an optimization over many combinations of inputs, it creates output page that has as its rows each strategy input combination and as its columns various trading performance measures such as Profit Factor, Total Net Profits, etc. An example of a simple filter would be to choose the strategy input optimization row in the in-sample section that had the highest Net Profit or perhaps a row that had the best Profit Factor with their associated strategy inputs. Unfortunately it was found that this type of simple metric performance filter very rarely produces good out-of-sample results. More complicated metric filters can produce good out-of-sample results minimizing spurious price movement biases in the selection of strategy inputs.

Here is a combination *filter* that is used in this paper with good out-of-sample results. High profit values (PF) in the in-sample section usually mean poor performance in the out-of-sample section. This is a kind of reversion to the mean. So in the in-sample section we eliminate all strategy input rows that have a PWFO metric $PF > 4$. In addition we wish to limit the number losing trades in a row in the 4 day IS period to 3 or less ($LR < 3$). The PWFO generates the metric **mWb**. This metric is the in-sample **Median of Winning Trade Bars**. For this metric we take the bars for each winning trade in the in-sample section and find the median winning bars for all winning trades. We use the median rather than the average because a few outliers can distort the average. Let us choose the 50 rows that contain the Largest(Top) **mWb** values from the rows that are left from the $PF - LR$ screen. This particular filter will now leave 50 cases or rows in the in-sample section that satisfy the above filter conditions. Suppose for this filter, within the 50 in-sample rows that are left, we want the row that has the maximum PWFO metric **m(ru-p)** in the in-sample section. **m(ru-p) = Median of All Trades(Maximum Trade Runup Minus Final Trade Profit)**. This metric measures the difference between the maximum profit (trade runup) of each trade and the final profit of the trade. **m(ru-p)** is the median of this difference for all trades for the given input variables. The closer the final trade profit is to the maximum trade profit, the better the performance of the input variable. Thus we would want the median to be as small as possible. Again we use the median for this statistic, because we do not want the statistic distorted by a few outlier trades. This would produce a filter named **t50mWb|p<4|lr3-m(ru-p)**. This in-sample filter leaves only one row in the PWFO in-sample section with its associated strategy inputs and out-of-sample net profit in the out-of-sample section. This particular **t50mWb|p<4|lr3-m(ru-p)** filter finds the strategy inputs parameters in each of the 308 in-sample sections and applies these inputs to the out-of-sample section. Using the filter in-sample strategy inputs on the 308 out-of-sample sections, the average out-of-sample performance is calculated. In addition many other important out-of-sample performance statistics for this filter are calculated and summarized. **Figure 3** shows such a filter computer run along with a small sample of other filter combinations that are constructed in a similar manner. Row 3 of the sample output in **Figure 3** shows the results of the filter discussed above.

A total of 7688 filters were examined. More on this below on how that number of filters combinations effect the probability that the filter chosen was due to chance or not.

Bootstrap Probability of Filter Results.

Using modern "Bootstrap" techniques, we can calculate the probability of obtaining our filter's total out-of-sample *net* profits by chance. Here is how the bootstrap technique is applied. Suppose as an example, we have 100 files of in-sample/out-of-sample data. A mirror random filter is created. Instead of picking an out-of-sample net profit (OSNP) from a filter row as before, the mirror filter picks a *random* row's OSNP in each of the 100 files. We repeat this random picking in each of the 100 files 5000 times. Each of the 5000 mirror filters will choose a random row's OSNP of their own in each of the 100 files.. At the end, each of the 5000 mirror filters will have 100 *random* OSNP's picked from the rows of the 100 files. The sum of the 100 random OSNP picks for each mirror filter will generate a random total out-of-sample net profit (tOnpNet) or final random equity. The average and standard deviation of the 5000 mirror filter's different random tOnPnets will allow us to calculate the chance probability of our above chosen filter's tOnpNet. Thus given the mirror filter's bootstrap random tOnpNet average and standard deviation, we can calculate the probability of obtaining our chosen filter's tOnpNet by pure chance alone. Figure 3 lists the 5000 mirror filter's bootstrap average for our 308 out-of-sample files of **(\$3473)** with a bootstrap standard deviation of **\$9581**. The probability of obtaining our filters net profit of **\$31,453** is 1.33×10^{-4} which is **3.64** standard deviations from the bootstrap average. For our filter, in row 3 in Figure 3, the expected number of cases that we could obtain by pure chance that would match or exceed the **\$31453** is $7688 \times 1.33 \times 10^{-4} = 1.02$ where **7688** is the total number of different filters we looked at in this run. This number is greater than 1 and means that if we chose 7688 random boot filters 1 out of the 7688, on average, would produce profits greater than our chosen filter. While this is a bit disconcerting because I like to see that number less than 1, it's close so I don't think that it is meaningful.

Results

Table 1 below presents a table of the 308 in-sample and out-of-sample windows, the **Filter** selected, strategy inputs and the daily out-of-sample profit/loss results using the filter described above.

Figure 1 presents a graph of the equity curve generated by using the filter on the 308 days ending 8/11/14 – 10/30/15 (note the starting 8/1/14 was part of the first 4 day in-sample period plus the number of MaxBarsBack and the OOS weekday after the weekend was 8/11/14). The equity curves is plotted from Equity and Net Equity columns in Table 1. Plotted on the equity curves is the 2nd Order Polynomial curve. The blue line is the equity curve without commissions and the red dots on the blue line are new highs in equity. The brown line is the equity curve with commissions and the green dots are the new highs in net equity. The black line is the ES Daily Closing prices superimposed on the Equity Chart.

Discussion of Strategy Performance

In Figure 3 Row 3 of the spreadsheet filter output are some statistics that are of interest for our filter. An interesting statistic is **Blw**. Blw is the maximum number of days the OSNP equity curve failed to make a new high. Blw is 26 days for this filter. This means that 26 trading days was the longest time that the equity for this strategy failed to make a new equity high.

To see the effect of walk forward analysis, take a look at **Table 1**. Notice how the input parameters *N*, *vup*, *vdn* take sudden jumps from high to low and back. This is the walk forward process quickly adapting to changing volatility conditions in the in-sample sample. In addition, notice how often *N* changes from 4 to 16. When the data gets very noisy with a lot of spurious price movements, the lookback period, *N*, has to be higher. During other times when the noise level is not as much *N* can be lower to get onboard a trend faster.

In Figure 1, which presents a graph of the equity curve using the filter on the 308 days of out-of-sample data, notice how the equity curve follows the 2nd order polynomial trend line with an R^2 of 0.96. This R^2 dropped to 0.94 for the net equity curve. In addition sharp drops of the Daily E-mini did not produce sharp drops in the equity.

Using this filter, the strategy was able to generate \$31453 net equity after commissions and slippage of \$25 trading one ES contract for 308 days. This period of time from 8/1/14 to 10/30/15 was a volatile market. Yet the LSQV strategy was able to adapt quite well. From Table 1, the largest losing day was -\$2538 on the Monday of 8/24/15. The largest drawdown was --\$2726 from the day ending on 8/26/15 to 9/3/15. However this drawdown only lasted 6 trading days and completely recovered and made a new equity high in 14 trading days. The longest time between new equity highs was 26 days.

In observing Table 1 we can see that this strategy and filter made trades from a low of 0 or no trades/day to a high of 10 trades/day with an average of 1.7 trades/day with a medium of 1 trade/day. For the no trade days, the inputs found by the filter in the in-sample section generated no trades in the out-of-sample section.

Given 23 hour trading of the ES, restricting the strategy to trade only from 830am to 3:00pm CT caused the strategy to miss many profitable trends opportunities when Asia and then Europe opened trading in the early morning. Further research will include the A.M. time zones.

Disclaimer

The strategies, methods and indicators presented here are given purely for educational purposes and to facilitate the research and study of trading methods in the financial markets. **Hypothetical out-of-sample test results are no guarantee of future profits.** Please be aware that the positive performance presented here is based upon hypothetical trading and can in no way give any assurances or claim that the strategy and methods presented here will produce profits in the future and in fact may create financial losses.

References

1. Meyers, Dennis (2013), "The British Pound Cubed, Redux", Technical Analysis of Stocks & Commodities, Volume 31: January
2. Meyers, Dennis (2005) "The Polynomial Velocity System Applied To E-Mini 1min Bars using Walk Forward, Out-Of-Sample Analysis", Working Paper Sept/2005, www.meyersanalytics.com/articles.php

**Figure 1 Graph of Least Squares Velocity Strategy Net Equity Applying the Walk Forward Filter Each Day
On ES 5min Bar Prices 8/11/2014 to 10/30/2015**

Note: The blue line is the equity curve without commissions and the red dots on the blue line are new highs in equity. The brown line is the equity curve with commissions and the green dots are the new highs in net equity. The black line is the ES Daily Closing prices superimposed on the Equity Chart.

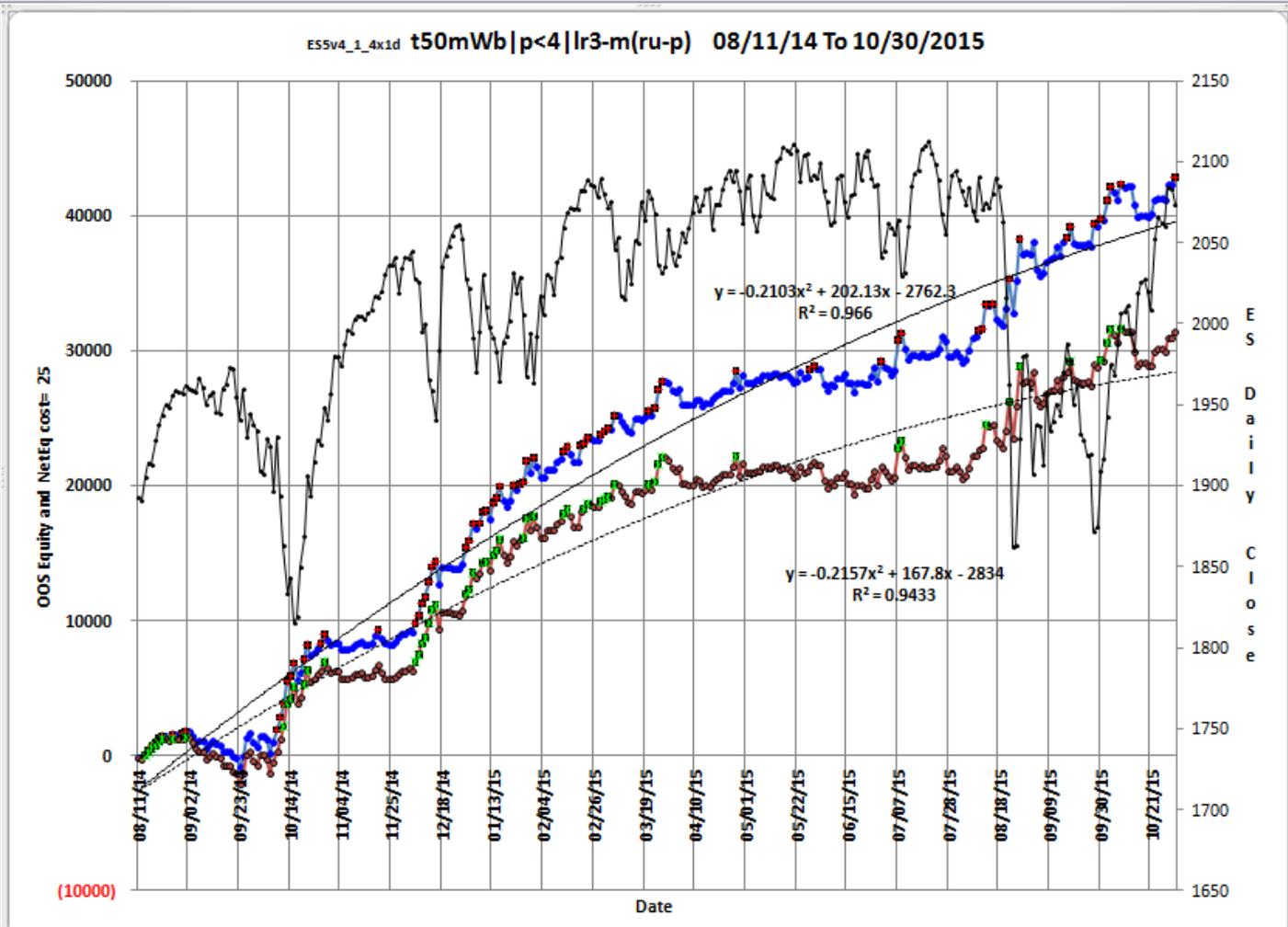


Figure 3 Partial output of the Walk Forward Metric Performance Explorer (WFME)
ES-Mini 1 min bars Least Squares Velocity System

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
1	ES5v4-1-4x1d	s08/11/14	e10/30/15	#308	AnyTnp				a(3473)	s9581	f7688										c=\$25	
2	Filter-Metric	tOnp	aOnp	aOTrd	aO#T	B0	%P	t	std	LLp	eqDD	olr	#	eqTrn	eqV^2	eqR2	Dev^2	Blw	BE	eff	tOnpNet	Prob
3	t50mWb p<4 lr3-m(ru-p)	42803	157	94.3	1.7	(0.1)	60	3.69	701	-2538	-2726	7	273	137	73	95	2742	26	80.1	0.29	31453	1.33E-04
4	t50mWb p<4-m(ru-p)	38377	139	81.1	1.7	0.0	59	3.37	686	-2538	-3326	7	276	113	66	95	2323	40	97.3	0.31	26552	8.62E-04
5	b50nT p<3-eq2R2	32102	147	113	1.3	1.4	57	3.67	593	-1413	-3440	7	218	99	159	89	3028	51	64.9	0.63	25002	1.48E-03
6	b50nT p<3 lr3-eq2R2	32102	147	113	1.3	1.4	57	3.67	593	-1413	-3440	7	218	99	159	89	3028	51	64.9	0.63	25002	1.48E-03
7	b50nT p<2 lr3-tLb	30462	159	122.3	1.3	0.1	59	3.31	665	-2563	-2713	4	191	101	69	96	1822	39	69.6	1.6	24237	1.91E-03

The WFME Filter Output Columns are defined as follows:

Row 1 ES1LSqV4dx1dxt is the strategy abbreviation, First OOS Day End Date(8/6/14), Last OOS Day End Date(7/31/15), **Number of days**(#308) **a**=average of bootstrap random picks. **s**= standard deviation of bootstrap random picks. **f**=number of different filters examined. **c**= slippage and round trip trade cost(c=\$25).

Filter = The filter that was run. Row 3 filter **t50mWb|p<4|lr3-m(ru-p)**

The **t50mWb|p<4|lr3-m(ru-p)** filter produced the following average 308 day statistics on row 3.

tOnp = Total out-of-sample(oos) net profit for these 308 days.

aOnp = Average oos net profit for the 308 days

aOTrd = Average oos profit per trade

aO#T = Average number of oos trades per day

B0 = The 308 day trend of the out-of-sample daily profits

%P = The percentage of oos days that were profitable

t = The student t statistic for the 308 daily oos profits. The higher the t statistic the higher the probability that this result was not due to pure chance

std = The standard deviation of the 308 daily oos profits

LLp = The largest losing oos period(day)

eqDD = The oos equity drawdown

olr = The largest number of losing oos days in a row

= The number of days this filter produced a daily result. Note for some days there can be no strategy inputs that satisfy a given filter's criteria.

eqTrn = The straight line trend of the oos gross profit equity curve in \$/day.

eqV^2 = The ending velocity of 2nd order polynomial that is fit to the equity curve

eqR2 = The correlation coefficient(r^2) of a straight line fit to the equity curve

Dev² = A measure of equity curve smoothness. The square root of the average [(equity curve minus a straight line)²]

Blw = The maximum number of days the oos equity curve failed to make a new high.

BE = Break even days. Assuming the average and standard deviation are from a normal distribution, this is the number of days you would have to trade to have a 98% probability that your oos equity is above zero.

eff = Efficiency. The average daily out-of-sample profit divided by the average daily in-sample profit.

tOnpNet = Total out-of-sample net profit(tOnpNet) minus the total trade cost.
$$tOnpNet = tOnp - (\text{Number of trade days}) * aOnT * \text{Cost}.$$

Prob = the probability that the filter's tOnpNet was due to pure chance.

**Table 1 Walk Forward Out-Of-Sample Performance Summary
ES-Mini 5 min bars Least Square Velocity Strategy**

ES-5 min bars 8/5/2014 - 10/30/20115 using the below filter on each in-sample segment. The input values *N*, *vup*, and *vdn* are the values found from applying the filter to the in-sample sample's optimization run.

In-sample Section Filter: t50mWb|p<4|lr3-m(p-rd)

Where:

osnp = Daily Out-of-sample net profit from strategy inputs chosen by In-sample Section filter

ont = The number of trades in the out-of-sample day from strategy inputs chosen by In-sample Section filter.

Equity = running sum of the daily out-of-sample profits(Osnp)

NetEq = running sum of daily out-of-sample profits minus \$25*Ont

Note: Blank rows indicate that no out-of-sample trades were made that day

In-Sample Days	OOS Day	Osnp	OnT	Equity	NetOsnp	NetEq	len	vup	vdn	xop	xt	mult	ES Close	ES Chg	
08/05/14	08/08/14	8/11/2014	(100)	1	(100)	(125)	(125)	10	0.5	2.5	0	1500	0.5359	1893	
08/06/14	08/11/14	8/12/2014	(63)	1	(163)	(88)	(213)	16	0.25	1.5	0	1500	0.5359	1891	(2.00)
08/07/14	08/12/14	8/13/2014	238	1	75	213	0	12	1	2	0	1500	0.5359	1905.25	14.25
08/08/14	08/13/14	8/14/2014	263	1	338	238	238	12	0.5	1.5	0	1500	0.5359	1914	8.75
08/11/14	08/14/14	8/15/2014	350	3	688	275	513	10	0.5	0.75	0	1500	0.5359	1913	(1.00)
08/12/14	08/15/14	8/18/2014	275	1	963	250	763	10	0.5	0.75	0	1500	0.5359	1928	15.00
08/13/14	08/18/14	8/19/2014	288	1	1251	263	1026	16	0.5	2	0	1500	0.5359	1937.75	9.75
08/14/14	08/19/14	8/20/2014	213	1	1464	188	1214	16	0.25	2	0	1500	0.5359	1943.75	6.00
08/15/14	08/20/14	8/21/2014	0	0	1464	0	1214	4	1.5	2	0	1500	0.5359	1950	6.25
08/18/14	08/21/14	8/22/2014	(63)	1	1401	(88)	1126	4	1.75	1.5	0	1500	0.5359	1948.25	(1.75)
08/19/14	08/22/14	8/25/2014	163	2	1564	113	1239	14	0.75	0.25	0	1500	0.5359	1955.5	7.25
08/20/14	08/25/14	8/26/2014	(25)	1	1539	(50)	1189	4	0.25	1.5	0	1500	0.5359	1959	3.50
08/21/14	08/26/14	8/27/2014	0	0	1539	0	1189	4	0.75	1.5	0	1500	0.5359	1957.5	(1.50)
08/22/14	08/27/14	8/28/2014	63	2	1602	13	1202	4	0.5	1.5	0	1500	0.5359	1957.25	(0.25)
08/25/14	08/28/14	8/29/2014	188	1	1790	163	1365	10	0.75	2	0	1500	0.5359	1962	4.75
08/27/14	09/01/14	9/2/2014	0	0	1790	0	1365	8	0.75	2.25	0	1500	0.5359	1960.25	(1.75)
08/28/14	09/02/14	9/3/2014	(375)	1	1415	(400)	965	6	0.75	1.25	0	1500	0.5359	1959.25	(1.00)
08/29/14	09/03/14	9/4/2014	(388)	3	1027	(463)	502	6	0.25	1.25	0	1500	0.5359	1958.25	(1.00)
09/01/14	09/04/14	9/5/2014	88	10	1115	(162)	340	4	0.75	0.25	0	1500	0.5359	1966.5	8.25
09/02/14	09/05/14	9/8/2014	0	0	1115	0	340	10	0.75	1.5	0	1500	0.5359	1961	(5.50)
09/03/14	09/08/14	9/9/2014	(550)	2	565	(600)	(260)	14	0.5	1.5	0	1500	0.5359	1950.25	(10.75)
09/04/14	09/09/14	9/10/2014	300	1	865	275	15	10	0.5	1.5	0	1500	0.5359	1955.5	5.25
09/05/14	09/10/14	9/11/2014	250	1	1115	225	240	8	0.25	1.5	0	1500	0.5359	1957.5	2.00
09/08/14	09/11/14	9/12/2014	(250)	1	865	(275)	(35)	4	0.25	2.5	0	1500	0.5359	1945.25	(12.25)
09/09/14	09/12/14	9/15/2014	(63)	1	802	(88)	(123)	4	1.75	0.75	0	1500	0.5359	1944.5	(0.75)
09/10/14	09/15/14	9/16/2014	(450)	5	352	(575)	(698)	10	1.25	0.25	0	1500	0.5359	1960	15.50
09/11/14	09/16/14	9/17/2014	0	1	352	(25)	(723)	8	2.25	0.75	0	1500	0.5359	1962.25	2.25
09/12/14	09/17/14	9/18/2014	0	0	352	0	(723)	8	2.25	0.75	0	1500	0.5359	1973	10.75
09/15/14	09/18/14	9/19/2014	(375)	1	(23)	(400)	(1123)	6	1.5	1.75	0	1500	0.5359	1972.25	(0.75)
09/16/14	09/19/14	9/22/2014	(100)	4	(123)	(200)	(1323)	8	0.25	1	0	1500	0.5359	1954.75	(17.50)
09/17/14	09/22/14	9/23/2014	(700)	2	(823)	(750)	(2073)	6	0.75	1.25	0	1500	0.5359	1940.75	(14.00)
09/18/14	09/23/14	9/24/2014	838	1	15	813	(1260)	6	1.25	1	0	1500	0.5359	1959.5	18.75
09/19/14	09/24/14	9/25/2014	1325	1	1340	1300	40	8	1.25	0.25	0	1500	0.5359	1930	(29.50)
09/22/14	09/25/14	9/26/2014	338	1	1678	313	353	4	1.5	3.5	0	1500	0.5359	1944.5	14.50
09/23/14	09/26/14	9/29/2014	(663)	1	1015	(688)	(335)	6	2.25	1	0	1500	0.5359	1938	(6.50)
09/24/14	09/29/14	9/30/2014	(300)	1	715	(325)	(660)	4	0.25	3.5	0	1500	0.5359	1934	(4.00)
09/25/14	09/30/14	10/1/2014	750	1	1465	725	65	4	2.5	1.5	0	1500	0.5359	1909.25	(24.75)
09/26/14	10/01/14	10/2/2014	38	2	1503	(12)	53	10	1.75	1.5	0	1500	0.5359	1907	(2.25)

In-Sample Days		OOS Day	Osnp	OnT	Equity	NetOsnp	NetEq	len	vup	vdn	xop	xt	mult	ES Close	ES Chg
09/29/14	10/02/14	10/3/2014	(313)	2	1190	(363)	(310)	12	2	0.5	0	1500	0.5359	1928.75	21.75
09/30/14	10/03/14	10/6/2014	(950)	2	240	(1000)	(1310)	6	2.25	2	0	1500	0.5359	1924.5	(4.25)
10/01/14	10/06/14	10/7/2014	813	1	1053	788	(522)	6	2.5	2.5	0	1500	0.5359	1896.5	(28.00)
10/02/14	10/07/14	10/8/2014	863	1	1916	838	316	6	2.5	2.5	0	1500	0.5359	1930.25	33.75
10/03/14	10/08/14	10/9/2014	925	1	2841	900	1216	4	3.25	3.25	0	1500	0.5359	1893.5	(36.75)
10/06/14	10/09/14	10/10/2014	963	1	3804	938	2154	4	3.5	0.25	0	1500	0.5359	1862.75	(30.75)
10/07/14	10/10/14	10/13/2014	1700	1	5504	1675	3829	10	3.25	0.25	0	1500	0.5359	1834	(28.75)
10/08/14	10/13/14	10/14/2014	400	1	5904	375	4204	16	3	0.75	0	1500	0.5359	1843.25	9.25
10/09/14	10/14/14	10/15/2014	950	2	6854	900	5104	14	2.25	1.75	0	1500	0.5359	1815.25	(28.00)
10/10/14	10/15/14	10/16/2014	(1238)	1	5616	(1263)	3841	12	3.5	1	0	1500	0.5359	1819	3.75
10/13/14	10/16/14	10/17/2014	563	2	6179	513	4354	14	3	0.25	0	1500	0.5359	1849.5	30.50
10/14/14	10/17/14	10/20/2014	950	1	7129	925	5279	16	0.25	3.25	0	1500	0.5359	1868.5	19.00
10/15/14	10/20/14	10/21/2014	1063	1	8192	1038	6317	4	0.25	3.5	0	1500	0.5359	1906.5	38.00
10/16/14	10/21/14	10/22/2014	(775)	1	7417	(800)	5517	4	0.25	3	0	1500	0.5359	1893.5	(13.00)
10/17/14	10/22/14	10/23/2014	263	2	7680	213	5730	16	0.25	1	0	1500	0.5359	1914.5	21.00
10/20/14	10/23/14	10/24/2014	375	1	8055	350	6080	16	0.25	1.75	0	1500	0.5359	1928.25	13.75
10/21/14	10/24/14	10/27/2014	250	1	8305	225	6305	6	0.25	3	0	1500	0.5359	1925.5	(2.75)
10/22/14	10/27/14	10/28/2014	663	1	8968	638	6943	10	0.25	2.75	0	1500	0.5359	1948.5	23.00
10/23/14	10/28/14	10/29/2014	(375)	1	8593	(400)	6543	10	1	2.5	0	1500	0.5359	1940.75	(7.75)
10/24/14	10/29/14	10/30/2014	(325)	1	8268	(350)	6193	8	1.75	1.5	0	1500	0.5359	1957	16.25
10/27/14	10/30/14	10/31/2014	113	1	8381	88	6281	6	2.25	2.25	0	1500	0.5359	1980	23.00
10/28/14	10/31/14	11/3/2014	0	0	8381	0	6281	10	2.25	1.75	0	1500	0.5359	1979.5	(0.50)
10/29/14	11/03/14	11/4/2014	(488)	1	7893	(513)	5768	10	1.75	1.75	0	1500	0.5359	1974	(5.50)
10/30/14	11/04/14	11/5/2014	0	1	7893	(25)	5743	8	1.25	2	0	1500	0.5359	1987.25	13.25
10/31/14	11/05/14	11/6/2014	0	0	7893	0	5743	10	1.25	1.25	0	1500	0.5359	1996.5	9.25
11/03/14	11/06/14	11/7/2014	100	1	7993	75	5818	6	0.25	2	0	1500	0.5359	1994.5	(2.00)
11/04/14	11/07/14	11/10/2014	275	1	8268	250	6068	14	0.25	1.5	0	1500	0.5359	2002.5	8.00
11/05/14	11/10/14	11/11/2014	63	1	8331	38	6106	4	0.75	1.75	0	1500	0.5359	2005	2.50
11/06/14	11/11/14	11/12/2014	88	2	8419	38	6144	6	0.5	1.25	0	1500	0.5359	2004.5	(0.50)
11/07/14	11/12/14	11/13/2014	(163)	5	8256	(288)	5856	8	0.25	1.25	0	1500	0.5359	2002.5	(2.00)
11/10/14	11/13/14	11/14/2014	(50)	1	8206	(75)	5781	8	0.25	2	0	1500	0.5359	2006.5	4.00
11/11/14	11/14/14	11/17/2014	188	1	8394	163	5944	4	0.25	2.5	0	1500	0.5359	2008.25	1.75
11/12/14	11/17/14	11/18/2014	488	1	8882	463	6407	6	0.25	1.5	0	1500	0.5359	2017	8.75
11/13/14	11/18/14	11/19/2014	425	2	9307	375	6782	12	0.25	1.5	0	1500	0.5359	2015.75	(1.25)
11/14/14	11/19/14	11/20/2014	(600)	1	8707	(625)	6157	4	1.5	1.75	0	1500	0.5359	2020.5	4.75
11/17/14	11/20/14	11/21/2014	(388)	1	8319	(413)	5744	4	1	3	0	1500	0.5359	2030.25	9.75
11/18/14	11/21/14	11/24/2014	(25)	1	8294	(50)	5694	4	1	3	0	1500	0.5359	2036	5.75
11/19/14	11/24/14	11/25/2014	0	0	8294	0	5694	6	1.25	2.75	0	1500	0.5359	2036	0.00
11/20/14	11/25/14	11/26/2014	175	1	8469	150	5844	4	0.25	3	0	1500	0.5359	2040.75	4.75
11/25/14	11/28/14	12/1/2014	338	4	8807	238	6082	4	0.25	1.75	0	1500	0.5359	2019.25	(21.50)
11/26/14	12/01/14	12/2/2014	225	1	9032	200	6282	16	1	1	0	1500	0.5359	2034.5	15.25
11/27/14	12/02/14	12/3/2014	0	0	9032	0	6282	16	1.5	1	0	1500	0.5359	2041	6.50
11/28/14	12/03/14	12/4/2014	250	1	9282	225	6507	4	1.25	3.5	0	1500	0.5359	2040.5	(0.50)
12/01/14	12/04/14	12/5/2014	(163)	1	9119	(188)	6319	4	2	1	0	1500	0.5359	2044.5	4.00
12/02/14	12/05/14	12/8/2014	688	2	9807	638	6957	4	1.5	1.5	0	1500	0.5359	2028	(16.50)
12/03/14	12/08/14	12/9/2014	575	2	10382	525	7482	8	0.25	2.25	0	1500	0.5359	2026	(2.00)
12/04/14	12/09/14	12/10/2014	913	3	11295	838	8320	4	2.5	1.5	0	1500	0.5359	1995	(31.00)
12/05/14	12/10/14	12/11/2014	450	2	11745	400	8720	4	2.75	2	0	1500	0.5359	1999.5	4.50
12/08/14	12/11/14	12/12/2014	1113	1	12858	1088	9808	4	2.75	0.25	0	1500	0.5359	1966	(33.50)
12/09/14	12/12/14	12/15/2014	1125	6	13983	975	10783	4	2.75	0.25	0	1500	0.5359	1958.75	(7.25)
12/10/14	12/15/14	12/16/2014	425	3	14408	350	11133	10	2.25	1.25	0	1500	0.5359	1940.5	(18.25)
12/11/14	12/16/14	12/17/2014	(1700)	1	12708	(1725)	9408	16	3.25	1	0	1500	0.5359	1983.75	43.25
12/12/14	12/17/14	12/18/2014	1300	1	14008	1275	10683	16	2.5	2	0	1500	0.5359	2035.5	51.75
12/15/14	12/18/14	12/19/2014	0	0	14008	0	10683	16	2.75	3	0	1500	0.5359	2042.5	7.00
12/16/14	12/19/14	12/22/2014	0	0	14008	0	10683	16	1	2.75	0	1500	0.5359	2048	5.50

In-Sample Days		OOS Day	Osnp	OnT	Equity	NetOsnp	NetEq	len	vup	vdn	xop	xt	mult	ES Close	ES Chg
12/17/14	12/22/14	12/23/2014	(138)	1	13870	(163)	10520	16	0.5	1.75	0	1500	0.5359	2054.5	6.50
12/22/14	12/25/14	12/26/2014	13	1	13883	(12)	10508	4	1.25	1.25	0	1500	0.5359	2059.75	5.25
12/23/14	12/26/14	12/29/2014	(38)	1	13845	(63)	10445	4	1.25	1.25	0	1500	0.5359	2061.25	1.50
12/24/14	12/29/14	12/30/2014	325	1	14170	300	10745	4	1.25	0.75	0	1500	0.5359	2052.25	(9.00)
12/25/14	12/30/14	12/31/2014	1250	1	15420	1225	11970	16	1.5	0.25	0	1500	0.5359	2028	(24.25)
12/29/14	01/01/15	1/2/2015	500	7	15920	325	12295	6	1	0.25	0	1500	0.5359	2021.75	(6.25)
12/30/14	01/02/15	1/5/2015	1263	1	17183	1238	13533	10	1.75	0.5	0	1500	0.5359	1991.5	(30.25)
12/31/14	01/05/15	1/6/2015	(300)	2	16883	(350)	13183	10	2	2.5	0	1500	0.5359	1970	(21.50)
01/01/15	01/06/15	1/7/2015	313	1	17196	288	13471	8	3.25	3.25	0	1500	0.5359	1995	25.00
01/02/15	01/07/15	1/8/2015	863	1	18059	838	14309	6	2.75	3	0	1500	0.5359	2030.5	35.50
01/05/15	01/08/15	1/9/2015	75	1	18134	50	14359	6	1.75	3	0	1500	0.5359	2010.75	(19.75)
01/06/15	01/09/15	1/12/2015	(600)	2	17534	(650)	13709	8	1.75	3.25	0	1500	0.5359	1998	(12.75)
01/07/15	01/12/15	1/13/2015	1188	2	18722	1138	14847	16	3	0.25	0	1500	0.5359	1991.5	(6.50)
01/08/15	01/13/15	1/14/2015	388	2	19110	338	15185	4	3.25	1.75	0	1500	0.5359	1983	(8.50)
01/09/15	01/14/15	1/15/2015	838	1	19948	813	15998	4	3.5	0.25	0	1500	0.5359	1964.5	(18.50)
01/12/15	01/15/15	1/16/2015	(1075)	1	18873	(1100)	14898	4	3.5	0.25	0	1500	0.5359	1988.5	24.00
01/14/15	01/19/15	1/20/2015	(463)	3	18410	(538)	14360	4	0.75	3	0	1500	0.5359	1992.25	3.75
01/15/15	01/20/15	1/21/2015	450	1	18860	425	14785	8	0.25	3.5	0	1500	0.5359	2002	9.75
01/16/15	01/21/15	1/22/2015	1188	1	20048	1163	15948	6	0.25	3.25	0	1500	0.5359	2032	30.00
01/19/15	01/22/15	1/23/2015	(338)	1	19710	(363)	15585	10	0.25	1.5	0	1500	0.5359	2019.5	(12.50)
01/20/15	01/23/15	1/26/2015	425	1	20135	400	15985	8	0.25	3	0	1500	0.5359	2029	9.50
01/21/15	01/26/15	1/27/2015	150	1	20285	125	16110	12	2.5	0.25	0	1500	0.5359	2005.5	(23.50)
01/22/15	01/27/15	1/28/2015	1513	2	21798	1463	17573	12	1.5	0.25	0	1500	0.5359	1967	(38.50)
01/23/15	01/28/15	1/29/2015	(813)	2	20985	(863)	16710	6	2	0.25	0	1500	0.5359	1994	27.00
01/26/15	01/29/15	1/30/2015	1088	3	22073	1013	17723	6	2.25	0.25	0	1500	0.5359	1964	(30.00)
01/27/15	01/30/15	2/2/2015	(675)	4	21398	(775)	16948	6	3	0.25	0	1500	0.5359	1992.5	28.50
01/28/15	02/02/15	2/3/2015	(750)	2	20648	(800)	16148	10	3	0.25	0	1500	0.5359	2017.5	25.00
01/29/15	02/03/15	2/4/2015	13	1	20661	(12)	16136	4	3	3.5	0	1500	0.5359	2005.5	(12.00)
01/30/15	02/04/15	2/5/2015	563	1	21224	538	16674	4	3	3.25	0	1500	0.5359	2030.5	25.00
02/02/15	02/05/15	2/6/2015	0	0	21224	0	16674	4	2.75	3.25	0	1500	0.5359	2028.5	(2.00)
02/03/15	02/06/15	2/9/2015	0	0	21224	0	16674	8	1.5	3.25	0	1500	0.5359	2018	(10.50)
02/04/15	02/09/15	2/10/2015	588	1	21812	563	17237	6	1.75	2.75	0	1500	0.5359	2037.75	19.75
02/05/15	02/10/15	2/11/2015	163	1	21975	138	17375	6	1.5	2.75	0	1500	0.5359	2041.25	3.50
02/06/15	02/11/15	2/12/2015	575	1	22550	550	17925	16	1	2	0	1500	0.5359	2059.5	18.25
02/09/15	02/12/15	2/13/2015	363	1	22913	338	18263	4	0.25	1.75	0	1500	0.5359	2069	9.50
02/11/15	02/16/15	2/17/2015	(525)	1	22388	(550)	17713	6	1.75	1.25	0	1500	0.5359	2071.5	2.50
02/12/15	02/17/15	2/18/2015	(638)	3	21750	(713)	17000	4	1.25	1.5	0	1500	0.5359	2071	(0.50)
02/13/15	02/18/15	2/19/2015	25	1	21775	0	17000	8	0.75	1.75	0	1500	0.5359	2070.75	(0.25)
02/16/15	02/19/15	2/20/2015	1200	2	22975	1150	18150	4	0.5	2.25	0	1500	0.5359	2082.5	11.75
02/17/15	02/20/15	2/23/2015	163	1	23138	138	18288	12	0.25	2	0	1500	0.5359	2082.25	(0.25)
02/18/15	02/23/15	2/24/2015	375	1	23513	350	18638	12	0.25	1.5	0	1500	0.5359	2089.25	7.00
02/19/15	02/24/15	2/25/2015	(25)	1	23488	(50)	18588	14	0.25	1.75	0	1500	0.5359	2085.75	(3.50)
02/20/15	02/25/15	2/26/2015	(138)	1	23350	(163)	18425	12	0.75	1.5	0	1500	0.5359	2085.5	(0.25)
02/23/15	02/26/15	2/27/2015	0	0	23350	0	18425	12	0.75	1.5	0	1500	0.5359	2078.25	(7.25)
02/24/15	02/27/15	3/2/2015	413	1	23763	388	18813	12	0.75	1.5	0	1500	0.5359	2089.5	11.25
02/25/15	03/02/15	3/3/2015	225	2	23988	175	18988	16	0.25	1.25	0	1500	0.5359	2080.25	(9.25)
02/26/15	03/03/15	3/4/2015	225	2	24213	175	19163	6	1.25	1	0	1500	0.5359	2072	(8.25)
02/27/15	03/04/15	3/5/2015	(38)	1	24175	(63)	19100	10	1.25	0.25	0	1500	0.5359	2075.25	3.25
03/02/15	03/05/15	3/6/2015	1013	1	25188	988	20088	10	1.25	0.25	0	1500	0.5359	2046.25	(29.00)
03/03/15	03/06/15	3/9/2015	0	0	25188	0	20088	4	1.5	3.25	0	1500	0.5359	2053.25	7.00
03/04/15	03/09/15	3/10/2015	(438)	2	24750	(488)	19600	4	0.25	2.5	0	1500	0.5359	2017.5	(35.75)
03/05/15	03/10/15	3/11/2015	(375)	1	24375	(400)	19200	8	1	2	0	1500	0.5359	2015	(2.50)
03/06/15	03/11/15	3/12/2015	(275)	3	24100	(350)	18850	6	1.5	0.75	0	1500	0.5359	2039.5	24.50
03/09/15	03/12/15	3/13/2015	(88)	2	24012	(138)	18712	6	1.5	1.75	0	1500	0.5359	2025.25	(14.25)
03/10/15	03/13/15	3/16/2015	950	1	24962	925	19637	4	2	3.25	0	1500	0.5359	2051.5	26.25

In-Sample Days		OOS Day	Osnp	OnT	Equity	NetOsnp	NetEq	len	vup	vdn	xop	xt	mult	ES Close	ES Chg
03/11/15	03/16/15	3/17/2015	0	0	24962	0	19637	4	2	3.25	0	1500	0.5359	2049.25	(2.25)
03/12/15	03/17/15	3/18/2015	(50)	2	24912	(100)	19537	6	1.5	2.5	0	1500	0.5359	2075.5	26.25
03/13/15	03/18/15	3/19/2015	238	1	25150	213	19750	6	1.5	0.75	0	1500	0.5359	2064.5	(11.00)
03/16/15	03/19/15	3/20/2015	400	2	25550	350	20100	12	1	0.5	0	1500	0.5359	2082.25	17.75
03/17/15	03/20/15	3/23/2015	(350)	1	25200	(375)	19725	4	2	2	0	1500	0.5359	2077.75	(4.50)
03/18/15	03/23/15	3/24/2015	588	1	25788	563	20288	10	3.5	0.25	0	1500	0.5359	2068	(9.75)
03/19/15	03/24/15	3/25/2015	1325	1	27113	1300	21588	6	1.75	0.75	0	1500	0.5359	2036.75	(31.25)
03/20/15	03/25/15	3/26/2015	600	3	27713	525	22113	16	0.75	0.5	0	1500	0.5359	2031.5	(5.25)
03/23/15	03/26/15	3/27/2015	0	0	27713	0	22113	14	1.25	0.75	0	1500	0.5359	2035.5	4.00
03/24/15	03/27/15	3/30/2015	(113)	2	27600	(163)	21950	14	1.5	0.25	0	1500	0.5359	2058.5	23.00
03/25/15	03/30/15	3/31/2015	(538)	5	27062	(663)	21287	16	0.5	0.5	0	1500	0.5359	2043.75	(14.75)
03/26/15	03/31/15	4/1/2015	(163)	2	26899	(213)	21074	10	1	1.25	0	1500	0.5359	2036	(7.75)
03/27/15	04/01/15	4/2/2015	263	1	27162	238	21312	10	1	1.25	0	1500	0.5359	2042.5	6.50
03/31/15	04/03/15	4/6/2015	(1150)	1	26012	(1175)	20137	8	3	1.25	0	1500	0.5359	2056.25	13.75
04/01/15	04/06/15	4/7/2015	0	0	26012	0	20137	4	2.75	3	0	1500	0.5359	2050.75	(5.50)
04/02/15	04/07/15	4/8/2015	(13)	1	25999	(38)	20099	12	2.75	0.25	0	1500	0.5359	2059	8.25
04/03/15	04/08/15	4/9/2015	0	0	25999	0	20099	4	3	3	0	1500	0.5359	2068.75	9.75
04/06/15	04/09/15	4/10/2015	413	1	26412	388	20487	6	0.25	2.75	0	1500	0.5359	2078.5	9.75
04/07/15	04/10/15	4/13/2015	(50)	2	26362	(100)	20387	8	0.25	1	0	1500	0.5359	2069.5	(9.00)
04/08/15	04/13/15	4/14/2015	(400)	3	25962	(475)	19912	8	0.25	1	0	1500	0.5359	2074	4.50
04/09/15	04/14/15	4/15/2015	125	1	26087	100	20012	10	0.25	1	0	1500	0.5359	2082.75	8.75
04/10/15	04/15/15	4/16/2015	13	1	26100	(12)	20000	12	0.25	1.5	0	1500	0.5359	2083.75	1.00
04/13/15	04/16/15	4/17/2015	338	3	26438	263	20263	4	2.5	0.25	0	1500	0.5359	2058.5	(25.25)
04/14/15	04/17/15	4/20/2015	288	1	26726	263	20526	6	0.25	3	0	1500	0.5359	2074	15.50
04/15/15	04/20/15	4/21/2015	88	1	26814	63	20589	16	2	0.5	0	1500	0.5359	2074	0.00
04/16/15	04/21/15	4/22/2015	250	1	27064	225	20814	16	1.25	1.25	0	1500	0.5359	2083.25	9.25
04/17/15	04/22/15	4/23/2015	0	0	27064	0	20814	16	1.5	0.75	0	1500	0.5359	2090	6.75
04/20/15	04/23/15	4/24/2015	0	0	27064	0	20814	4	3	1.75	0	1500	0.5359	2094.75	4.75
04/21/15	04/24/15	4/27/2015	600	1	27664	575	21389	4	3	0.25	0	1500	0.5359	2087.75	(7.00)
04/22/15	04/27/15	4/28/2015	825	2	28489	775	22164	4	0.75	1.25	0	1500	0.5359	2095	7.25
04/23/15	04/28/15	4/29/2015	(1175)	8	27314	(1375)	20789	4	0.75	1.5	0	1500	0.5359	2082	(13.00)
04/24/15	04/29/15	4/30/2015	950	3	28264	875	21664	4	2.5	0.25	0	1500	0.5359	2062	(20.00)
04/27/15	04/30/15	5/1/2015	(600)	2	27664	(650)	21014	12	2	0.25	0	1500	0.5359	2084.5	22.50
04/28/15	05/01/15	5/4/2015	13	1	27677	(12)	21002	8	0.25	3	0	1500	0.5359	2092.25	7.75
04/29/15	05/04/15	5/5/2015	0	0	27677	0	21002	16	1.75	2.5	0	1500	0.5359	2067	(25.25)
04/30/15	05/05/15	5/6/2015	163	1	27840	138	21140	4	3.25	0.25	0	1500	0.5359	2057.25	(9.75)
05/01/15	05/06/15	5/7/2015	0	0	27840	0	21140	16	1.75	2	0	1500	0.5359	2067.25	10.00
05/04/15	05/07/15	5/8/2015	350	1	28190	325	21465	14	1.5	2.25	0	1500	0.5359	2091.5	24.25
05/05/15	05/08/15	5/11/2015	0	0	28190	0	21465	8	1	1.75	0	1500	0.5359	2080.75	(10.75)
05/06/15	05/11/15	5/12/2015	(113)	2	28077	(163)	21302	8	1	3	0	1500	0.5359	2078	(2.75)
05/07/15	05/12/15	5/13/2015	263	2	28340	213	21515	8	3	0.5	0	1500	0.5359	2077.5	(0.50)
05/08/15	05/13/15	5/14/2015	0	0	28340	0	21515	8	3	0.5	0	1500	0.5359	2100.5	23.00
05/11/15	05/14/15	5/15/2015	(288)	1	28052	(313)	21202	10	2.25	0.25	0	1500	0.5359	2102	1.50
05/12/15	05/15/15	5/18/2015	150	1	28202	125	21327	12	0.5	1	0	1500	0.5359	2109	7.00
05/13/15	05/18/15	5/19/2015	0	0	28202	0	21327	8	1	1.5	0	1500	0.5359	2107.5	(1.50)
05/14/15	05/19/15	5/20/2015	(213)	2	27989	(263)	21064	14	1	0.75	0	1500	0.5359	2105.5	(2.00)
05/15/15	05/20/15	5/21/2015	(400)	1	27589	(425)	20639	16	1.5	0.25	0	1500	0.5359	2111	5.50
05/18/15	05/21/15	5/22/2015	125	1	27714	100	20739	16	1.5	0.25	0	1500	0.5359	2107.5	(3.50)
05/20/15	05/25/15	5/26/2015	750	1	28464	725	21464	10	1.75	0.75	0	1500	0.5359	2088	(19.50)
05/21/15	05/26/15	5/27/2015	(475)	2	27989	(525)	20939	8	1.25	0.25	0	1500	0.5359	2104	16.00
05/22/15	05/27/15	5/28/2015	138	1	28127	113	21052	16	0.25	3.25	0	1500	0.5359	2104.75	0.75
05/25/15	05/28/15	5/29/2015	488	1	28615	463	21515	10	2.25	0.25	0	1500	0.5359	2089	(15.75)
05/26/15	05/29/15	6/1/2015	238	1	28853	213	21728	16	0.75	3.25	0	1500	0.5359	2092.25	3.25
05/27/15	06/01/15	6/2/2015	(200)	1	28653	(225)	21503	6	2.75	0.75	0	1500	0.5359	2089.75	(2.50)
05/28/15	06/02/15	6/3/2015	0	0	28653	0	21503	4	2.25	2.25	0	1500	0.5359	2099	9.25

In-Sample Days		OOS Day	Osnp	OnT	Equity	NetOsnp	NetEq	len	vup	vdn	xop	xt	mult	ES Close	ES Chg
05/29/15	06/03/15	6/4/2015	(1088)	2	27565	(1138)	20365	8	0.25	1.75	0	1500	0.5359	2082	(17.00)
06/01/15	06/04/15	6/5/2015	(488)	3	27077	(563)	19802	6	2	1.5	0	1500	0.5359	2075.25	(6.75)
06/02/15	06/05/15	6/8/2015	463	1	27540	438	20240	14	2	0.25	0	1500	0.5359	2061.25	(14.00)
06/03/15	06/08/15	6/9/2015	(175)	1	27365	(200)	20040	4	2.25	0.25	0	1500	0.5359	2063	1.75
06/04/15	06/09/15	6/10/2015	663	2	28028	613	20653	16	1	0.25	0	1500	0.5359	2090	27.00
06/05/15	06/10/15	6/11/2015	(13)	1	28015	(38)	20615	16	2	0.25	0	1500	0.5359	2092.25	2.25
06/08/15	06/11/15	6/12/2015	363	1	28378	338	20953	4	1.75	2	0	1500	0.5359	2075.75	(16.50)
06/09/15	06/12/15	6/15/2015	(688)	3	27690	(763)	20190	10	1.5	0.25	0	1500	0.5359	2066.25	(9.50)
06/10/15	06/15/15	6/16/2015	0	0	27690	0	20190	6	2.25	1.25	0	1500	0.5359	2079.75	13.50
06/11/15	06/16/15	6/17/2015	(713)	2	26977	(763)	19427	16	1.75	0.5	0	1500	0.5359	2080	0.25
06/12/15	06/17/15	6/18/2015	600	1	27577	575	20002	4	3.5	1.75	0	1500	0.5359	2105.5	25.50
06/15/15	06/18/15	6/19/2015	0	0	27577	0	20002	4	1.5	2.5	0	1500	0.5359	2088.5	(17.00)
06/16/15	06/19/15	6/22/2015	(113)	1	27464	(138)	19864	4	3.5	1.75	0	1500	0.5359	2103.5	15.00
06/17/15	06/22/15	6/23/2015	0	0	27464	0	19864	4	3.5	1.5	0	1500	0.5359	2107.25	3.75
06/18/15	06/23/15	6/24/2015	675	1	28139	650	20514	14	3	0.25	0	1500	0.5359	2090.25	(17.00)
06/19/15	06/24/15	6/25/2015	588	1	28727	563	21077	14	0.75	0.25	0	1500	0.5359	2084.75	(5.50)
06/22/15	06/25/15	6/26/2015	(963)	3	27764	(1038)	20039	16	0.75	1.25	0	1500	0.5359	2086.5	1.75
06/23/15	06/26/15	6/29/2015	1438	1	29202	1413	21452	16	0.75	0.25	0	1500	0.5359	2041.25	(45.25)
06/24/15	06/29/15	6/30/2015	(375)	1	28827	(400)	21052	16	0.5	1.75	0	1500	0.5359	2045.25	4.00
06/25/15	06/30/15	7/1/2015	(200)	2	28627	(250)	20802	14	1.5	0.5	0	1500	0.5359	2061.75	16.50
06/26/15	07/01/15	7/2/2015	(363)	1	28264	(388)	20414	4	1.75	2.5	0	1500	0.5359	2059.5	(2.25)
06/30/15	07/03/15	7/6/2015	250	3	28514	175	20589	4	2.5	0.5	0	1500	0.5359	2055.25	(4.25)
07/01/15	07/06/15	7/7/2015	2250	2	30764	2200	22789	4	2.5	2.25	0	1500	0.5359	2064.5	9.25
07/02/15	07/07/15	7/8/2015	538	1	31302	513	23302	16	1.25	3.25	0	1500	0.5359	2030	(34.50)
07/03/15	07/08/15	7/9/2015	(1200)	1	30102	(1225)	22077	10	0.25	2.75	0	1500	0.5359	2032	2.00
07/06/15	07/09/15	7/10/2015	(775)	2	29327	(825)	21252	4	3	1.25	0	1500	0.5359	2059.75	27.75
07/07/15	07/10/15	7/13/2015	325	1	29652	300	21552	4	2.75	1.5	0	1500	0.5359	2085.25	25.50
07/08/15	07/13/15	7/14/2015	0	0	29652	0	21552	6	2.75	1.5	0	1500	0.5359	2092.75	7.50
07/09/15	07/14/15	7/15/2015	(113)	3	29539	(188)	21364	4	0.25	1.5	0	1500	0.5359	2095	2.25
07/10/15	07/15/15	7/16/2015	238	1	29777	213	21577	6	0.25	1.75	0	1500	0.5359	2107.75	12.75
07/13/15	07/16/15	7/17/2015	(200)	1	29577	(225)	21352	10	1	0.5	0	1500	0.5359	2109.5	1.75
07/14/15	07/17/15	7/20/2015	38	1	29615	13	21365	14	0.25	1.25	0	1500	0.5359	2112.75	3.25
07/15/15	07/20/15	7/21/2015	138	1	29753	113	21478	8	1	0.75	0	1500	0.5359	2105.25	(7.50)
07/16/15	07/21/15	7/22/2015	13	1	29766	(12)	21466	14	0.5	0.5	0	1500	0.5359	2098.75	(6.50)
07/17/15	07/22/15	7/23/2015	400	1	30166	375	21841	14	0.75	0.5	0	1500	0.5359	2089.25	(9.50)
07/20/15	07/23/15	7/24/2015	963	1	31129	938	22779	10	1	0.25	0	1500	0.5359	2068.25	(21.00)
07/21/15	07/24/15	7/27/2015	(438)	5	30691	(563)	22216	4	1.5	0.5	0	1500	0.5359	2055.25	(13.00)
07/22/15	07/27/15	7/28/2015	(1088)	1	29603	(1113)	21103	4	2.5	0.25	0	1500	0.5359	2078	22.75
07/23/15	07/28/15	7/29/2015	0	0	29603	0	21103	8	2	1.75	0	1500	0.5359	2092.25	14.25
07/24/15	07/29/15	7/30/2015	338	2	29941	288	21391	6	1	2.25	0	1500	0.5359	2094.5	2.25
07/27/15	07/30/15	7/31/2015	(300)	1	29641	(325)	21066	8	0.5	3.25	0	1500	0.5359	2089.25	(5.25)
07/28/15	07/31/15	8/3/2015	(488)	3	29153	(563)	20503	14	0.25	2.25	0	1500	0.5359	2081.75	(7.50)
07/29/15	08/03/15	8/4/2015	250	1	29403	225	20728	4	2	0.75	0	1500	0.5359	2073.75	(8.00)
07/30/15	08/04/15	8/5/2015	688	2	30091	638	21366	4	2	0.75	0	1500	0.5359	2084.5	10.75
07/31/15	08/05/15	8/6/2015	850	1	30941	825	22191	14	2	0.25	0	1500	0.5359	2070.25	(14.25)
08/03/15	08/06/15	8/7/2015	113	1	31054	88	22279	12	2.5	0.5	0	1500	0.5359	2064.25	(6.00)
08/04/15	08/07/15	8/10/2015	450	1	31504	425	22704	14	2	0.75	0	1500	0.5359	2090.5	26.25
08/05/15	08/10/15	8/11/2015	125	1	31629	100	22804	16	1	2	0	1500	0.5359	2070.5	(20.00)
08/06/15	08/11/15	8/12/2015	1788	2	33417	1738	24542	4	0.25	3	0	1500	0.5359	2075	4.50
08/07/15	08/12/15	8/13/2015	(88)	1	33329	(113)	24429	4	2.25	1.5	0	1500	0.5359	2071.25	(3.75)
08/10/15	08/13/15	8/14/2015	138	2	33467	88	24517	8	1	0.75	0	1500	0.5359	2080.25	9.00
08/11/15	08/14/15	8/17/2015	(1138)	1	32329	(1163)	23354	6	2.75	1	0	1500	0.5359	2090	9.75
08/12/15	08/17/15	8/18/2015	(200)	1	32129	(225)	23129	6	0.25	2	0	1500	0.5359	2084.75	(5.25)
08/13/15	08/18/15	8/19/2015	(250)	3	31879	(325)	22804	6	2.75	1	0	1500	0.5359	2063.5	(21.25)
08/14/15	08/19/15	8/20/2015	1250	1	33129	1225	24029	8	3	1.25	0	1500	0.5359	2016.25	(47.25)

In-Sample Days		OOS Day	Osnp	OnT	Equity	NetOsnp	NetEq	len	vup	vdn	xop	xt	mult	ES Close	ES Chg
08/17/15	08/20/15	8/21/2015	2188	1	35317	2163	26192	4	2.75	0.75	0	1500	0.5359	1962.25	(54.00)
08/18/15	08/21/15	8/24/2015	(2538)	5	32779	(2663)	23529	14	2.5	1.25	0	1500	0.5359	1862	(100.25)
08/19/15	08/24/15	8/25/2015	2463	4	35242	2363	25892	6	3.5	0.25	0	1500	0.5359	1863.5	1.50
08/20/15	08/25/15	8/26/2015	3050	4	38292	2950	28842	14	2.75	0.25	0	1500	0.5359	1928.75	65.25
08/21/15	08/26/15	8/27/2015	(1188)	3	37104	(1263)	27579	14	2.25	3.25	0	1500	0.5359	1980	51.25
08/24/15	08/27/15	8/28/2015	200	1	37304	175	27754	14	3.5	3	0	1500	0.5359	1980.5	0.50
08/25/15	08/28/15	8/31/2015	(125)	1	37179	(150)	27604	14	2.25	3	0	1500	0.5359	1960	(20.50)
08/26/15	08/31/15	9/1/2015	900	1	38079	875	28479	12	2	3.5	0	1500	0.5359	1906.75	(53.25)
08/27/15	09/01/15	9/2/2015	(2100)	2	35979	(2150)	26329	14	2.25	1.5	0	1500	0.5359	1937.75	31.00
08/28/15	09/02/15	9/3/2015	(413)	1	35566	(438)	25891	6	0.25	3.25	0	1500	0.5359	1936.75	(1.00)
08/31/15	09/03/15	9/4/2015	263	1	35829	238	26129	4	3	0.25	0	1500	0.5359	1912.5	(24.25)
09/02/15	09/07/15	9/8/2015	738	1	36567	713	26842	12	2	2.5	0	1500	0.5359	1956.5	44.00
09/03/15	09/08/15	9/9/2015	213	2	36780	163	27005	4	3	2.75	0	1500	0.5359	1933.5	(23.00)
09/04/15	09/09/15	9/10/2015	100	1	36880	75	27080	4	3	2.75	0	1500	0.5359	1939.75	6.25
09/07/15	09/10/15	9/11/2015	825	1	37705	800	27880	6	0.5	2.75	0	1500	0.5359	1950.25	10.50
09/08/15	09/11/15	9/14/2015	(713)	3	36992	(788)	27092	6	0.25	3	0	1500	0.5359	1944	(6.25)
09/09/15	09/14/15	9/15/2015	1050	1	38042	1025	28117	4	0.5	3	0	1500	0.5359	1970	26.00
09/10/15	09/15/15	9/16/2015	313	1	38355	288	28405	10	0.75	2	0	1500	0.5359	1988	18.00
09/11/15	09/16/15	9/17/2015	838	2	39193	788	29193	8	0.5	2.5	0	1500	0.5359	1977.25	(10.75)
09/14/15	09/17/15	9/18/2015	(1213)	4	37980	(1313)	27880	8	0.5	2.5	0	1500	0.5359	1950.5	(26.75)
09/15/15	09/18/15	9/21/2015	(75)	1	37905	(100)	27780	16	2.75	3	0	1500	0.5359	1963	12.50
09/16/15	09/21/15	9/22/2015	(113)	2	37792	(163)	27617	12	1.25	2.75	0	1500	0.5359	1932	(31.00)
09/17/15	09/22/15	9/23/2015	25	1	37817	0	27617	10	2	0.75	0	1500	0.5359	1928.5	(3.50)
09/18/15	09/23/15	9/24/2015	175	2	37992	125	27742	12	1.25	2.75	0	1500	0.5359	1918.75	(9.75)
09/21/15	09/24/15	9/25/2015	(313)	3	37679	(388)	27354	8	2.5	2.5	0	1500	0.5359	1919.25	0.50
09/22/15	09/25/15	9/28/2015	1725	1	39404	1700	29054	4	3.25	0.25	0	1500	0.5359	1872	(47.25)
09/23/15	09/28/15	9/29/2015	(150)	4	39254	(250)	28804	6	2.75	0.5	0	1500	0.5359	1874.5	2.50
09/24/15	09/29/15	9/30/2015	513	1	39767	488	29292	14	3.25	2.5	0	1500	0.5359	1908.75	34.25
09/25/15	09/30/15	10/1/2015	(38)	1	39729	(63)	29229	4	2	3.5	0	1500	0.5359	1916.75	8.00
09/28/15	10/01/15	10/2/2015	1413	2	41142	1363	30592	6	2	3	0	1500	0.5359	1943	26.25
09/29/15	10/02/15	10/5/2015	1000	1	42142	975	31567	4	2.5	3.5	0	1500	0.5359	1974.75	31.75
09/30/15	10/05/15	10/6/2015	(338)	1	41804	(363)	31204	6	0.25	2.75	0	1500	0.5359	1968.5	(6.25)
10/01/15	10/06/15	10/7/2015	(588)	2	41216	(638)	30566	16	2	0.25	0	1500	0.5359	1987.25	18.75
10/02/15	10/07/15	10/8/2015	1113	1	42329	1088	31654	16	0.25	3.5	0	1500	0.5359	2006.5	19.25
10/05/15	10/08/15	10/9/2015	(188)	1	42141	(213)	31441	16	1	1.5	0	1500	0.5359	2007.5	1.00
10/06/15	10/09/15	10/12/2015	50	1	42191	25	31466	16	0.5	3.25	0	1500	0.5359	2011	3.50
10/07/15	10/12/15	10/13/2015	0	0	42191	0	31466	6	2	2.75	0	1500	0.5359	1994	(17.00)
10/08/15	10/13/15	10/14/2015	(1350)	6	40841	(1500)	29966	16	0.25	1	0	1500	0.5359	1984	(10.00)
10/09/15	10/14/15	10/15/2015	(963)	2	39878	(1013)	28953	4	2.25	0.5	0	1500	0.5359	2019	35.00
10/12/15	10/15/15	10/16/2015	150	1	40028	125	29078	4	1.5	3	0	1500	0.5359	2025.5	6.50
10/13/15	10/16/15	10/19/2015	50	2	40078	0	29078	4	1.5	3	0	1500	0.5359	2027.5	2.00
10/14/15	10/19/15	10/20/2015	(163)	2	39915	(213)	28865	4	0.25	2.25	0	1500	0.5359	2020.5	(7.00)
10/15/15	10/20/15	10/21/2015	188	4	40103	88	28953	6	0.25	1.75	0	1500	0.5359	2008.5	(12.00)
10/16/15	10/21/15	10/22/2015	1025	1	41128	1000	29953	12	0.25	1.75	0	1500	0.5359	2053	44.50
10/19/15	10/22/15	10/23/2015	175	1	41303	150	30103	8	0.25	1.75	0	1500	0.5359	2066	13.00
10/20/15	10/23/15	10/26/2015	0	0	41303	0	30103	4	2	2.5	0	1500	0.5359	2062.25	(3.75)
10/21/15	10/26/15	10/27/2015	(113)	2	41190	(163)	29940	4	0.75	2.25	0	1500	0.5359	2060.5	(1.75)
10/22/15	10/27/15	10/28/2015	1113	3	42303	1038	30978	4	0.75	2	0	1500	0.5359	2084.5	24.00
10/23/15	10/28/15	10/29/2015	0	0	42303	0	30978	6	3	2	0	1500	0.5359	2083	(1.50)
10/26/15	10/29/15	10/30/2015	500	1	42803	475	31453	10	2.75	0.25	0	1500	0.5359	2073.75	(9.25)

Appendix: The Normalization Multiplier

What is the Multiplier ?

The Least Square Velocity, is the least square fit of a of a straight to a set of prices

If you are fitting the straight line to N prices then the “Best Fit” coefficients **a** and **b** can be solved for easily and are given by

$$a = [2(2N+1)/N(N-1)] \sum_1^N p(t) - [6/(N(N-1))] \sum_1^N t * p(t)$$

$$b = \text{Velocity} = [12/N(N^2 - 1)] \sum_1^N t * p(t) - [6/N(N-1)] \sum_1^N p(t)$$

Where **p(t)** is the price at point time point **t** and **N** is the number of prices we are using to calculate the coefficients. Here **p(1)** is the first price in the series and **p(N)** is the last price in the series.

One of the inputs to the calculation of Velocity is the **N** the number of lookback bars. When we plot the velocity we notice that the amplitude, and the maximum and minimum values of the velocity vary quite significantly with different **N** inputs.

Below is a table of the standard deviation of the 45828 calculated Velocity values for different **N**. We used 5 min bars of the E-Mini from 8/1/2014 to 10/30/2015 to generate this table.

LSqVelocity Multiplier to Scale Velocity N Range to One Std					
@ES.D	5min bars	DateRange	1130801	to	1151030
TotalBars	45828	iNorm=1			
N	Std	Std*Sqr(N)	1/Std		
4	1.00861	2.01723	0.49573		
6	0.777723	1.90502	0.524928		
8	0.658807	1.86339	0.536657		
10	0.581811	1.83985	0.543524		
12	0.527158	1.82613	0.547607		
14	0.485466	1.81645	0.550525		
16	0.452462	1.80985	0.552533		
ave	0.64172	1.86827	0.535929		

As one can see the Velocity Standard Deviation(SD) for N=4 is approximately 2 times the SD for N=16. This makes it difficult to find a set of vup and vdn that satisfy all N. We would like to find a multiplier of the Velocity that normalizes all the N SDs and ranges to one SD.

Fortunately the SDs for the different Ns for a Least Squares Velocity are proportional to \sqrt{N} . So if we multiply the Velocity by the \sqrt{N} , SDs of the Velocities for different N as shown in the table above (Std*Sqrt(N), are very close..

If we multiply all velocities by $0.5359 \cdot \sqrt{N}$ then the SDs of the velocities for all N will be normalized very close to 1. This allows us to do an optimization search for ranges of v_{up} and v_{dn} from 0.25 to 3.5 standard deviations for all N.